Math 220 (section AD?) Quiz 10 Fall 2013

Name Solutions

- You have 20 minutes
- No calculators
- Show sufficient work

1. (2 points) Precisely state *The Mean Value Theorem.*

Suppose the following:

① f is continuous on [a, b]

② f is differentiable on (a, b)

Then $f'(c) = \frac{f(b)-f(a)}{b-a}$

for some c in (a, b)

2. (2 points) Evaluate the definite integral. Simplify your final answer.

$$\int_{\sqrt{5}}^{\sqrt{21}} \frac{6x}{\sqrt{x^2+4}}\,dx = \int_9^{25} \frac{3du}{\sqrt{u}} = \int_9^{25} 3u^{-1/2}\,du$$

$$= 6u^{1/2}\Big|_9^{25}$$

$$= 6(25)^{1/2} - 6(9)^{1/2}$$

$$= 6\cdot 5 - 6\cdot 3$$

$$= 12$$

$u = x^2 + 4$

$\frac{du}{dx} = 2x \Rightarrow du = 2x\,dx$

$\Rightarrow 3du = 6x\,dx$

$x = \sqrt{5} \Rightarrow u = (\sqrt{5})^2 + 4 = 9$

$x = \sqrt{21} \Rightarrow u = (\sqrt{21})^2 + 4 = 25$

3. (2 points each) Evaluate the indefinite integrals.

(a) $\int \frac{6x+15}{9x^2+1}\,dx = \int \frac{6x}{9x^2+1}\,dx + \int \frac{15}{9x^2+1}\,dx$

$$= \frac{1}{3}\ln(9x^2+1) + 5\arctan(3x) + C$$

$\int \frac{6x}{9x^2+1}\,dx = \int \frac{\frac{1}{3}du}{u}$

$u = 9x^2+1$
$du = 18x\,dx$
$\frac{1}{3}du = 6x\,dx$

$= \frac{1}{3}\int \frac{1}{u}\,du$
$= \frac{1}{3}\ln|u| + C$
$= \frac{1}{3}\ln(9x^2+1) + C$

$\int \frac{15}{9x^2+1}\,dx = \int \frac{15}{(3x)^2+1}\,dx$

$w = 3x$
$dw = 3dx$
$5dw = 15dx$

$= \int \frac{5\,dw}{w^2+1}$
$= 5\arctan(w) + C$
$= 5\arctan(3x) + C$

(b) $\int 5x^{14}\left(x^5+4\right)^{100}\,dx$

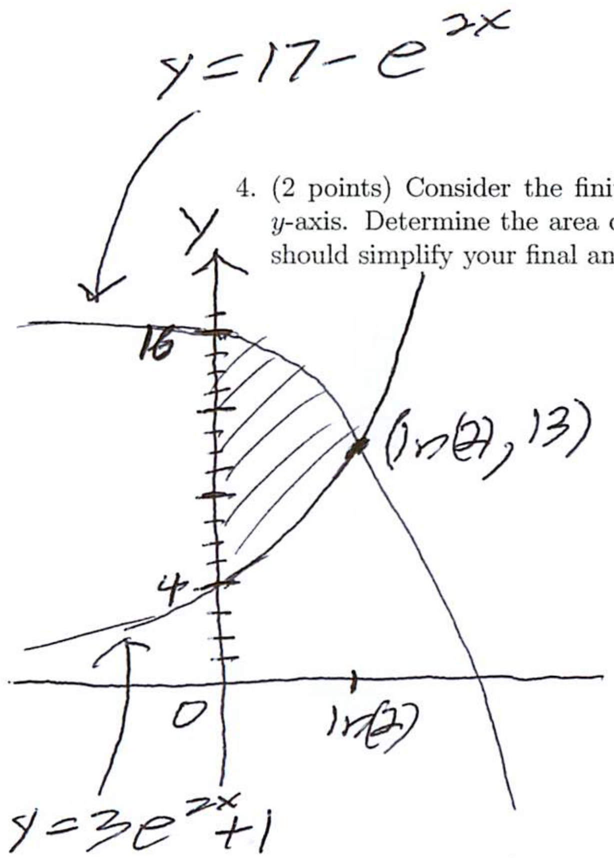
$= \int 5x^4 (x^5)^2 (x^5+4)^{100}\,dx$

$u = x^5+4 \Rightarrow u-4 = x^5$
$du = 5x^4\,dx$

$= \int (u-4)^2 u^{100}\,du$
$= \int (u^2-8u+16)u^{100}\,du$
$= \int (u^{102}-8u^{101}+16u^{100})\,du$
$= \frac{1}{103}u^{103} - \frac{8}{102}u^{102} + \frac{16}{101}u^{101} + C$

$$= \frac{1}{103}(x^5+4)^{103} - \frac{4}{51}(x^5+4)^{102} + \frac{16}{101}(x^5+4)^{101} + C$$

4. (2 points) Consider the finite region **R** bounded by $y = 3e^{2x} + 1$, $y = 17 - e^{2x}$ and the y-axis. Determine the area of **R** by evaluating a definite integral with respect to x. You should simplify your final answer.



intersection

$$3e^{2x}+1 = 17 - e^{2x}$$
$$4e^{2x} = 16$$
$$e^{2x} = 4$$
$$\ln(e^{2x}) = \ln(4) = 2\ln(2)$$
$$2x = 2\ln(2)$$
$$x = \ln(2)$$
$$\text{and } y = 17 - e^{2\ln(2)} = 17 - e^{\ln(4)} = 17 - 4 = 13$$

$$\text{area}(R) = \int_0^{\ln(2)} (y_{top} - y_{bottom})\,dx$$
$$= \int_0^{\ln(2)} \left((17 - e^{2x}) - (3e^{2x}+1)\right) dx$$
$$= \int_0^{\ln(2)} (16 - 4e^{2x})\,dx$$
$$= \int_0^{2\ln(2)} (16 - 4e^{u})\tfrac{1}{2}\,du$$
$$= \int_0^{2\ln(2)} (8 - 2e^{u})\,du$$
$$= (8u - 2e^{u})\Big|_0^{2\ln(2)} = (16\ln(2) - 2e^{2\ln(2)}) - (0 - 2)$$
$$= 16\ln(2) - 8 + 2$$
$$= 16\ln(2) - 6$$

$u = 2x$

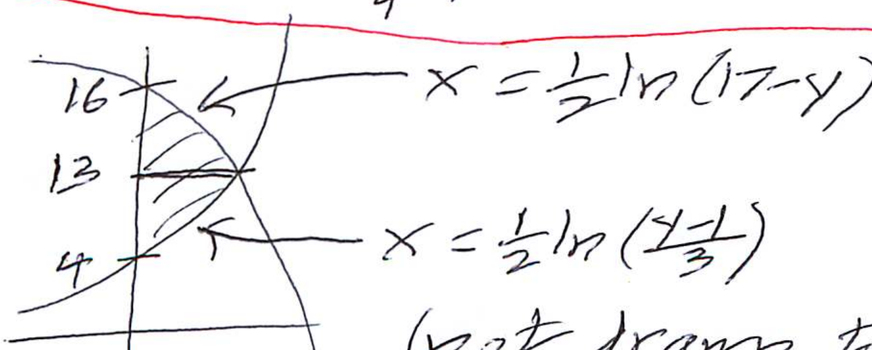
$du = 2\,dx$

$\frac{1}{2}du = dx$

x	$u = 2x$
0	0
$\ln(2)$	$2\ln(2)$

5. (1 bonus point) Set up, <u>but do not evaluate</u>, one or more definite integrals with respect to y to represent the area of **R** in the previous problem.

$$\text{area}(R) = \int_4^{13} \frac{1}{2}\ln\left(\frac{y-1}{3}\right) dy + \int_{13}^{16} \frac{1}{2}\ln(17-y)\,dy$$



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