Math 220 – Test 3 Information

The test will be given during your lecture period on Wednesday (December 5, 2012). No books, notes, scratch paper, calculators or other electronic devices are allowed. Bring a Student ID.

It may be helpful to look at

- `http://www.math.illinois.edu/~murphyrf/teaching/M220-F2012/` – *Daily Assignments* for summary of each lecture
- `https://compass2g.illinois.edu/` – homework solutions
- `http://www.math.illinois.edu/~murphyrf/teaching/M220/` – tests in my previous courses

- **Section 3.10 (Linear Approximation and Differentials)**
  - Be able to use a tangent line (or differentials) in order to approximate the value of a function near the point of tangency.
- **Section 4.2 (The Mean Value Theorem)**
  - Be able to precisely state *The Mean Value Theorem* and *Rolle's Theorem*.
  - Be able to decide when functions satisfy the conditions of these theorems. If a function does satisfy the conditions, then be able to find the value of c guaranteed by the theorems.
  - Be able to use *The Mean Value Theorem*, *Rolle's Theorem*, or earlier important theorems such as *The Intermediate Value Theorem* to prove some other fact. In the homework these often involved roots, solutions, x-intercepts, or intersection points.
- **Section 4.8 (Newton's Method)**
  - Understand the graphical basis for Newton's Method (that is, use the point where the tangent line crosses the x-axis as your next estimate for a root of a function).
  - Be able to apply Newton's Method to approximate roots, solutions, x-intercepts, or intersection points.
- **Section 4.9 (Antiderivatives)**
  - Know antiderivative formulas for 0, k (a constant), $\sin x$, $\cos x$, $\sec^2 x$, $\csc^2 x$, $\sec x \tan x$, $\csc x \cot x$, e^x, $\dfrac{1}{1+x^2}$, $\dfrac{1}{\sqrt{1-x^2}}$, x^n $(n \neq -1)$, $x^{-1} = \dfrac{1}{x}$.
  - Be able to find general antiderivatives for functions which are sums or differences of constants multiplied by the above formulas (you may need to simplify first).
  - Be able to solve a differential equation where values for the function or its first or second derivative are given.
  - Be able to apply these rules to problems involving acceleration, velocity, or position.
  - You should know that the acceleration due to gravity is approximately $-32\ ft/s^2$ or $-9.8\ m/s^2$ near the Earth's surface.

- **Section 5.1 (Areas and Distances)**
  - Use Riemann sums (left, right, or midpoint) to estimate area or total change in a quantity and state if your estimate is known to be an underestimate or overestimate. These sums will involve at most 8 subintervals.
  - Use limits of Riemann sums to find the exact area or total change in a quantity. Being able to do this with right Riemann sums will be sufficient for this test.
  - Understand sigma notation for sums and know the following sums.
    * $\sum_{k=1}^{n} C = C \cdot n$ (C is a constant).
    * $\sum_{k=1}^{n} k = \frac{n(n+1)}{2}$.
    * $\sum_{k=1}^{n} k^2 = \frac{n(n+1)(2n+1)}{6}$.
    * $\sum_{k=1}^{n} k^3 = \left(\frac{n(n+1)}{2}\right)^2$.
- **Section 5.2 (The Definite Integral)**
  - Understand the definition of a definite integral as $\int_a^b f(x)\,dx = \lim_{n\to\infty} \sum_{i=1}^{n} f(x_i^*)\Delta x$. Be able to more explicitly write out the appropriate limit for specific functions on given intervals. You may have to evaluate one such limit.
  - Know the relationship between a definite integral and area. This should be understood regardless of whether or not the graph of the function being integrated is above or below the x-axis.
  - Know the following properties of the definite integral.
    * $\int_b^a f(x)\,dx = -\int_a^b f(x)\,dx$
    * $\int_a^a f(x)\,dx = 0$
    * $\int_a^b c\,dx = c(b-a)$ where c is any constant
    * $\int_a^b [f(x)+g(x)]\,dx = \int_a^b f(x)\,dx + \int_a^b g(x)\,dx$
    * $\int_a^b cf(x)\,dx = c\int_a^b f(x)\,dx$ where c is any constant
    * $\int_a^b [f(x)-g(x)]\,dx = \int_a^b f(x)\,dx - \int_a^b g(x)\,dx$
    * $\int_a^c f(x)\,dx + \int_c^b f(x)\,dx = \int_a^b f(x)\,dx$
    * If $f(x) \geq 0$ for $a \leq x \leq b$, then $\int_a^b f(x)\,dx \geq 0$
    * If $f(x) \geq g(x)$ for $a \leq x \leq b$, then $\int_a^b f(x)\,dx \geq \int_a^b g(x)\,dx$
    * If $m \leq f(x) \leq M$ for $a \leq x \leq b$, then $m(b-a) \leq \int_a^b f(x)\,dx \leq M(b-a)$

- **Section 5.3 (The Fundamental Theorem of Calculus)**
  - Be able to precisely state *Part 1* and *Part 2* of the *The Fundamental Theorem of Calculus.*
  - When the conditions of the theorem hold, be able to use *Part 1* to find the derivative of functions which are defined in terms of integrals.
  - When the conditions of the theorem hold, be able to use *Part 2* to evaluate definite integrals.

- **Section 5.4 (Indefinite Integrals and the Net Change Theorem)**
  - Know indefinite integral formulas for 0, k (a constant), $\sin x$, $\cos x$, $\sec^2 x$, $\csc^2 x$, $\sec x \tan x$, $\csc x \cot x$, e^x, $\frac{1}{1+x^2}$, $\frac{1}{\sqrt{1-x^2}}$, x^n $(n \neq -1)$, $x^{-1} = \frac{1}{x}$.
  - Know that the definite integral of a rate of change gives the total change. Be able to use this *Net Change Theorem* for applied problems involving rates of change such as velocity, acceleration, growth rates, etc.

- **Section 5.5 (The Substitution Rule)**
  - Be able to solve a wide variety of definite or indefinite integrals using substitution.
  - Be able to more quickly evaluate definite integrals on the interval $[-a, a]$ given that the integrand is continuous and either even or odd on that interval.

- **Section 6.1 (Areas between Curves)**
  - Be able to find areas between curves. This may require breaking the area up into the sum of two or more definite integrals.
  - Be able to integrate with respect to x or with respect to y to determine these areas.

- **Section 6.2 (Volumes)**
  - Be able to find volumes for solids formed by revolving a region around any vertical or horizontal line.
  - Be able to find volumes for solids formed by building upon some base and having cross-sections which are rectangles, squares, triangles, semi-circles, etc.
  - Be able to integrate with respect to x or with respect to y to determine these volumes.

- **Section 6.3 (Volumes by Cylindrical Shells)**
  - Be able to find volumes for solids formed by revolving a region around any vertical or horizontal line.
  - Be able to integrate with respect to x or with respect to y to determine these volumes.

- **Section 6.5 (Average Value of a Function)**
  - Be able to find the average value of a function.
  - Know the graphical interpretation of the average value of a function.
  - Know *The Mean Value Theorem for Integrals.*

- **Section 7.2 (Trigonometric Integrals)**
  - Be able to use substitution to solve definite or indefinite integrals involving trigonometric functions.
  - Be able to use basic trigonometric definitions and identities to help evaluate these integrals. In particular be able to use
    * $\tan x = \dfrac{\sin x}{\cos x}$
    * $\cot x = \dfrac{\cos x}{\sin x}$
    * $\sec x = \dfrac{1}{\cos x}$
    * $\csc x = \dfrac{1}{\sin x}$
    * $\sin^2 x + \cos^2 x = 1$
    * $\tan^2 x + 1 = \sec^2 x$
    * $1 + \cot^2 x = \csc^2 x$
    * $\sin(2x) = 2\sin x \cos x$
    * $\cos(2x) = \cos^2 x - \sin^2 x = 2\cos^2 x - 1 = 1 - 2\sin^2 x$
    * $\cos^2 x = \dfrac{1}{2} + \dfrac{1}{2}\cos(2x)$
    * $\sin^2 x = \dfrac{1}{2} - \dfrac{1}{2}\cos(2x)$