Math 220 **Quiz 8 (take-home)** **Fall 2012**

Name SOLUTIONS

(circle your TA discussion section)

- ▷ **AD1**, TR 11:00-12:50, Hannah Kolb Spinoza
- ▷ **AD2**, TR 9:00-10:50, Ki Yeun Kim
- ▷ **AD3**, TR 1:00-2:50, Michael Santana
- ▷ **ADA**, TR 8:00-8:50, Ziying Pan
- ▷ **ADB**, TR 9:00-9:50, Ziying Pan
- ▷ **ADC**, TR 10:00-10:50, Lisa Hickok
- ▷ **ADD**, TR 11:00-11:50, Lisa Hickok
- ▷ **ADE**, TR 12:00-12:50, Andrew McConvey
- ▷ **ADF**, TR 1:00-1:50, Jian Liang
- ▷ **ADG**, TR 2:00-2:50, Derrek Yager
- ▷ **ADH**, TR 3:00-3:50, Lechao Xiao
- ▷ **ADI**, TR 4:00-4:50, Lechao Xiao
- ▷ **ADJ**, TR 9:00-9:50, Meghan Galiardi
- ▷ **ADK**, TR 10:00-10:50, Meghan Galiardi
- ▷ **ADL**, TR 11:00-11:50, Andrew McConvey
- ▷ **ADM**, TR 12:00-12:50, Benjamin Fulan
- ▷ **ADN**, TR 1:00-1:50, Benjamin Fulan
- ▷ **ADO**, TR 2:00-2:50, Jian Liang
- ▷ **ADP**, TR 3:00-3:50, Hongfei Tian
- ▷ **ADQ**, TR 4:00-4:50, Hongfei Tian
- ▷ **ADR**, TR 9:00-9:50, Noah Chartoff
- ▷ **ADS**, TR 12:00-12:50, Derrek Yager
- ▷ **ADT**, TR 2:00-2:50, Anna Weigandt
- ▷ **ADU**, TR 3:00-3:50, Anna Weigandt

- You may work with other MATH 220 students. However each student should write up solutions separately and independently – nobody should copy someone else's work.
- You may use your notes or the textbook.
- Computers are not allowed on any problem. You may use a calculator only for basic arithmetic.
- The quiz should be submitted to your TA at the beginning of your discussion section on Tuesday, November 6th.
- There is a higher expectation for the quality of your work on a take-home quiz. Everything should be written logically and legibly with sufficient work to justify each answer. Blank copies of the quiz are available on the course home page.
- Be sure that the pages are nicely stapled – do not just fold the corners.
- **Note to TAs and Tutors – you should not help students with these specific problems or go over solutions until after 5pm Tuesday.**

1. (2 points) The acceleration due to gravity near the surface of some planet is $-6\ m/s^2$. An object is shot upward from the surface of this planet and 10 seconds later it has fallen back to the surface. What is the velocity of this object 2 seconds after being shot?

(acc) $s'' = -6$

(vel) $s' = -6t + C$

(pos) $s = -3t^2 + Ct + D$

$s(0) = 0 \Rightarrow D = 0$

$s = -3t^2 + Ct$

$s(10) = 0 \Rightarrow -300 + 10C = 0 \Rightarrow C = 30$

$s = -3t^2 + 30t$ and $s' = -6t + 30$

$s'(2) = -12 + 30 = 18\ m/s$

2. (2 points) Find a formula for $g(t)$ given that $g''(t) = 3\cos t - 8e^t + 24t - 6$, $g'(0) = 2$ and $g(0) = 4$.

$g'(t) = 3\sin t - 8e^t + 12t^2 - 6t + C$

$g'(0) = 2 \Rightarrow 3\sin(0) - 8e^0 + 12(0)^2 - 6(0) + C = 2$

$\Rightarrow -8 + C = 2$

$\Rightarrow C = 10$

$g'(t) = 3\sin t - 8e^t + 12t^2 - 6t + 10$

$g(t) = -3\cos t - 8e^t + 4t^3 - 3t^2 + 10t + D$

$g(0) = 4 \Rightarrow -3\cos(0) - 8e^0 + 4(0)^3 - 3(0)^2 + 10(0) + D = 4$

$\Rightarrow -3 - 8 + D = 4$

$\Rightarrow D = 15$

$g(t) = -3\cos t - 8e^t + 4t^3 - 3t^2 + 10t + 15$

3. (2 points) Suppose that f is continuous at all real numbers, $\int_1^3 f(x)\,dx = 4$ and $\int_1^5 f(x)\,dx = 13$.

What is the value of $\int_3^5 (2f(x)+10)\,dx$?

$$\int_3^5 f(x)\,dx = \int_3^1 f(x)\,dx + \int_1^5 f(x)\,dx$$
$$= -\int_1^3 f(x)\,dx + \int_1^5 f(x)\,dx$$
$$= -4 + 13 = 9$$

$$\int_3^5 (2f(x)+10)\,dx = 2\int_3^5 f(x)\,dx + \int_3^5 10\,dx$$
$$= 2\cdot 9 + 10\cdot 2$$
$$= 38$$

4. (2 points) Evaluate the following limit.

$$\lim_{n\to\infty} \sum_{k=1}^{n} \left(\frac{20}{n^4} + \frac{15k^2}{n^3} + \frac{8k}{n^2} + \frac{7}{n}\right) = \lim_{n\to\infty}\left(\sum_{k=1}^{n}\frac{20}{n^4} + \sum_{k=1}^{n}\frac{15k^2}{n^3} + \sum_{k=1}^{n}\frac{8k}{n^2} + \sum_{k=1}^{n}\frac{7}{n}\right)$$

$$= \lim_{n\to\infty}\left(\frac{20}{n^4}\sum_{k=1}^{n} 1 + \frac{15}{n^3}\sum_{k=1}^{n} k^2 + \frac{8}{n^2}\sum_{k=1}^{n} k + \frac{7}{n}\sum_{k=1}^{n} 1\right)$$

$$= \lim_{n\to\infty}\left(\frac{20}{n^4}\cdot n + \frac{15}{n^3}\cdot\frac{n(n+1)(2n+1)}{6} + \frac{8}{n^2}\cdot\frac{n(n+1)}{2} + \frac{7}{n}\cdot n\right)$$

$$= \lim_{n\to\infty}\left(\frac{20}{n^3} + \frac{30n^3+45n^2+15n}{6n^3} + \frac{8n^2+8n}{2n^2} + 7\right)$$

$$= \lim_{n\to\infty}\left(\frac{20}{n^3} + \frac{30n^3}{6n^3} + \frac{45n^2}{6n^3} + \frac{15n}{6n^3} + \frac{8n^2}{2n^2} + \frac{8n}{2n^2} + 7\right)$$

$$= \lim_{n\to\infty}\left(\frac{20}{n^3} + 5 + \frac{15}{2n} + \frac{5}{2n^2} + 4 + \frac{4}{n} + 7\right)$$

$$= 0 + 5 + 0 + 0 + 4 + 0 + 7$$

$$= 16$$

5. (2 points) The area between the x-axis and the graph of $f(x) = x^2$ on the interval $[3, 7]$ can be written as a limit of Riemann sums in many different ways. I have shown how to do this for two of the six ways indicated below. Fill in the missing information for the remaining limits so that the only variables appearing are n and k. Do not evaluate these limits.

(a) Using a limit of right Riemann sums,

$$AREA = \lim_{n\to\infty} \sum_{k=1}^{n} \left[\left(3 + k \cdot \frac{4}{n}\right)^2 \cdot \frac{4}{n} \right]$$

(b) Using a limit of right Riemann sums,

$$AREA = \lim_{n\to\infty} \sum_{k=0}^{n-1} \left[\left(3 + (K+1) \cdot \frac{4}{n}\right)^2 \cdot \frac{4}{n} \right]$$

(c) Using a limit of left Riemann sums,

$$AREA = \lim_{n\to\infty} \sum_{k=1}^{n} \left[\left(3 + (K-1) \cdot \frac{4}{n}\right)^2 \cdot \frac{4}{n} \right]$$

(d) Using a limit of left Riemann sums,

$$AREA = \lim_{n\to\infty} \sum_{k=0}^{n-1} \left[\left(3 + K \cdot \frac{4}{n}\right)^2 \cdot \frac{4}{n} \right]$$

(e) Using a limit of midpoint Riemann sums,

$$AREA = \lim_{n\to\infty} \sum_{k=1}^{n} \left[\left(3 + (K-0.5) \cdot \frac{4}{n}\right)^2 \cdot \frac{4}{n} \right]$$

(f) Using a limit of midpoint Riemann sums,

$$AREA = \lim_{n\to\infty} \sum_{k=0}^{n-1} \left[\left(3 + (k+0.5) \cdot \frac{4}{n}\right)^2 \cdot \frac{4}{n} \right]$$