Math 220 **Quiz 7 (take-home)** **Fall 2012**

Name Solutions

(circle your TA discussion section)

- ▷ **AD1**, TR 11:00-12:50, Hannah Kolb Spinoza
- ▷ **AD2**, TR 9:00-10:50, Ki Yeun Kim
- ▷ **AD3**, TR 1:00-2:50, Michael Santana
- ▷ **ADA**, TR 8:00-8:50, Ziying Pan
- ▷ **ADB**, TR 9:00-9:50, Ziying Pan
- ▷ **ADC**, TR 10:00-10:50, Lisa Hickok
- ▷ **ADD**, TR 11:00-11:50, Lisa Hickok
- ▷ **ADE**, TR 12:00-12:50, Andrew McConvey
- ▷ **ADF**, TR 1:00-1:50, Jian Liang
- ▷ **ADG**, TR 2:00-2:50, Derrek Yager
- ▷ **ADH**, TR 3:00-3:50, Lechao Xiao
- ▷ **ADI**, TR 4:00-4:50, Lechao Xiao
- ▷ **ADJ**, TR 9:00-9:50, Meghan Galiardi
- ▷ **ADK**, TR 10:00-10:50, Meghan Galiardi
- ▷ **ADL**, TR 11:00-11:50, Andrew McConvey
- ▷ **ADM**, TR 12:00-12:50, Benjamin Fulan
- ▷ **ADN**, TR 1:00-1:50, Benjamin Fulan
- ▷ **ADO**, TR 2:00-2:50, Jian Liang
- ▷ **ADP**, TR 3:00-3:50, Hongfei Tian
- ▷ **ADQ**, TR 4:00-4:50, Hongfei Tian
- ▷ **ADR**, TR 9:00-9:50, Noah Chartoff
- ▷ **ADS**, TR 12:00-12:50, Derrek Yager
- ▷ **ADT**, TR 2:00-2:50, Anna Weigandt
- ▷ **ADU**, TR 3:00-3:50, Anna Weigandt

- You may work with other MATH 220 students. However each student should write up solutions separately and independently – nobody should copy someone else's work.
- You may use your notes or the textbook.
- Computers are not allowed on any problem. You may use a calculator only for basic arithmetic.
- The quiz should be submitted to Mr. Murphy at the beginning of your official lecture period on Friday, October 19th.
- There is a higher expectation for the quality of your work on a take-home quiz. Everything should be written logically and legibly with sufficient work to justify each answer. Blank copies of the quiz are available on the course home page.
- Be sure that the pages are nicely stapled – do not just fold the corners.
- **Note to TAs and Tutors – you should not help students with these specific problems or go over solutions until after 5pm Friday.**

1. (3 points) Evaluate $\lim_{x \to 0} (1-5x)^{8/x}$

This limit is of indeterminate form 1^{∞} as $x \to 0^+$

(or $1^{-\infty}$ as $x \to 0^-$)

$$\lim_{x \to 0} (1-5x)^{8/x} = \lim_{x \to 0} e^{\ln\left((1-5x)^{8/x}\right)}$$

$$= e^{\lim_{x \to 0} \ln\left((1-5x)^{8/x}\right)}$$

(OKAY since e^u is continuous at all u)

$$= e^{\lim_{x \to 0} \frac{8}{x} \cdot \ln(1-5x)}$$

$$= e^{\lim_{x \to 0} \frac{8\ln(1-5x)}{x}} \quad \begin{matrix} \to 0 \\ \to 0 \end{matrix}$$

$$= e^{\lim_{x \to 0} \frac{8 \cdot \frac{1}{1-5x} \cdot (-5)}{1}}$$

(by L'Hospital's Rule)

$$= e^{\lim_{x \to 0} \frac{-40}{1-5x}}$$

$$= e^{-40}$$

2. (4 points) Suppose that $f(x)$ is a function whose first derivative is given below.

$$f'(x) = \frac{2e^x\left(x^2+25\right)\left(x^2-16\right)(x-3)^8}{\ln\left(x^2+100\right)}$$

Find all critical numbers of $f(x)$. At each critical number, state whether $f(x)$ has a local maximum, local minimum or neither at that point.

f' exists for all x since the domain of f' is all real numbers.

$$f' = 0 \text{ at } \underbrace{x=-4,\ x=4, \text{ and } x=3}_{\text{critical numbers}}$$

values of f'

+++ 0 — — — 0 - - - 0 +++

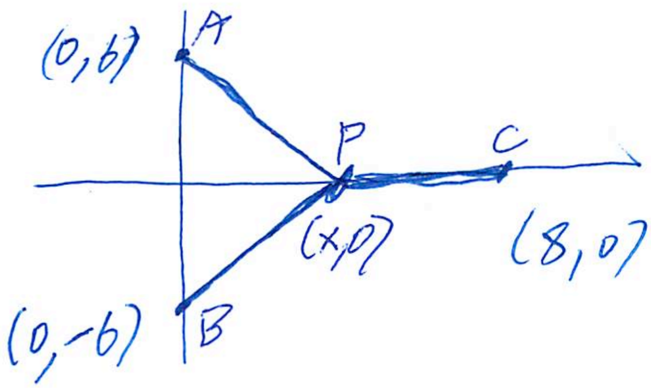
-4 $\quad$ 3 $\quad$ 4

f has a local max at $x=-4$

f has neither a local max nor a local min at $x=3$

f has a local min at $x=4$

3. (3 points) Suppose that point A has coordinates $(0, 6)$, point B has coordinates $(0, -6)$, and point C has coordinates $(8, 0)$. Determine the coordinates for the point P on the x-axis for which the sum of the distances from P to each of the three points A, B and C is as small as possible.



P is on the x-axis, so its coordinates are $(x,0)$. It is also clear that $0 \le x \le 8$.

SUM OF DISTANCES = Dist(P,A) + Dist(P,B) + Dist(P,C)

$$S = \sqrt{x^2+36} + \sqrt{x^2+36} + 8 - x$$

$$S = 2\sqrt{x^2+36} + 8 - x$$

we wish to minimize S on $[0,8]$

$$S' = 2 \cdot \tfrac{1}{2}(x^2+36)^{-1/2} \cdot 2x + 0 - 1$$

$$= \frac{2x}{\sqrt{x^2+36}} - 1$$

S' exists for all x so its only critical points satisfy $S' = 0$.

$$0 = \frac{2x}{\sqrt{x^2+36}} - 1 \Rightarrow 2x = \sqrt{x^2+36} \Rightarrow 4x^2 = x^2+36$$

$\Rightarrow 3x^2 = 36 \Rightarrow x^2 = 12 \Rightarrow x = \pm\sqrt{12}$ but only $x = \sqrt{12}$ satisfies the original equation

Since S is continuous on $[0,12]$ we use the closed interval method.

$S(0) = 2\sqrt{0^2+36} + 8 - 0 = 20$

$S(\sqrt{12}) = 2\sqrt{(\sqrt{12})^2+36} + 8 - \sqrt{12} = 6\sqrt{3} + 8 < 20$

$S(8) = 2\sqrt{8^2+36} + 8 - 8 = 20$

min. occurs at $x = \sqrt{12} = 2\sqrt{3}$

P is at $(2\sqrt{3}, 0)$