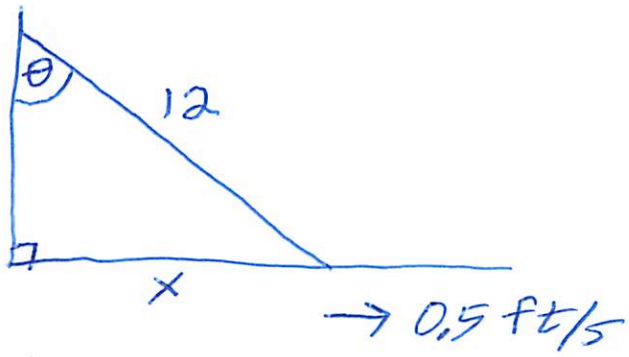
Name Solutions

- You have 15 minutes
- No calculators
- Show sufficient work

1. (3 points) A ladder 12 feet long rests against a vertical wall. If the bottom of the ladder slides away from the wall at a rate of 0.5 feet per second, how quickly in radians per second is the angle between the ladder and the wall increasing when the bottom of the ladder is 5 feet from the wall?



Given: $\frac{dx}{dt} = 0.5 \text{ ft/s}$

Find: $\left.\frac{d\theta}{dt}\right|_{x=5\text{ft}}$

$\sin\theta = \frac{x}{12}$

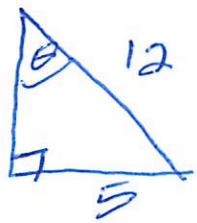
$\frac{d}{dt}(\sin\theta) = \frac{d}{dt}\left(\frac{x}{12}\right)$

$\cos\theta \cdot \frac{d\theta}{dt} = \frac{1}{12} \cdot \frac{dx}{dt}$

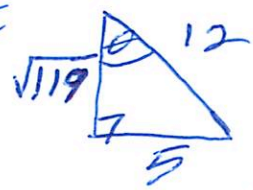
$\frac{\sqrt{119}}{12} \cdot \frac{d\theta}{dt} = \frac{1}{12} \cdot 0.5 = \frac{1}{24}$

$\frac{d\theta}{dt} = \frac{1}{24} \cdot \frac{12}{\sqrt{119}} = \frac{1}{2\sqrt{119}} = \frac{\sqrt{119}}{238}$ radians/second

At moment $x = 5$ ft we have



using Pythagorean Theorem, we get



so $\cos\theta = \frac{\sqrt{119}}{12}$

2. (4 points) A rock is thrown vertically upward from the surface of a planet. The rock's height above the planet's surface is given by the equation $s = t(24 - 1.2t)$, where t is measured in seconds and s is measured in meters.

(a) Find a formula for the rock's velocity at time t.

$s = t(24 - 1.2t) = 24t - 1.2t^2$

velocity is $s' = 24 - 2.4t$

(b) What is the maximum height reached by the rock?

set $s' = 0$

$0 = 24 - 2.4t$

$2.4t = 24$

$t = \frac{24}{2.4} = 10$ seconds

values of s'

$+++ \; 0 \; ---$

$\longrightarrow t$

10

max height is $s(10) = 10(24 - 1.2(10))$

$= 120$ meters

3. (3 points) Determine a formula for P as a function of t given that $8P' + 2P = 0$ and $P(0) = 5$. Hint: You may recognize the solution more quickly if you first solve the given equation for P'.

$8P' + 2P = 0 \Rightarrow P' = -\frac{1}{4}P$ (equivalently, $\frac{dP}{dt} = -\frac{1}{4}P$)

$P = Ce^{-\frac{1}{4}t}$

$P(0) = 5 \Rightarrow 5 = Ce^{-\frac{1}{4}\cdot 0} \Rightarrow C = 5$

$P = 5e^{-\frac{1}{4}t}$