

Name

SOLUTIONS

• You have 15 minutes

• No calculators

• Show sufficient work

1. (3 points) Compute  $\frac{dw}{dq}$  given that  $w^2q^3 = \sin(w^2) + q^5$ .

$$\frac{d}{dq}(w^2q^3) = \frac{d}{dq}(\sin(w^2) + q^5)$$

$$\frac{d}{dq}(w^2) \cdot q^3 + w^2 \cdot \frac{d}{dq}(q^3) = \cos(w^2) \cdot \frac{d}{dq}(w^2) + 5q^4$$

$$2w \cdot \frac{dw}{dq} \cdot q^3 + w^2 \cdot 3q^2 = \cos(w^2) \cdot 2w \cdot \frac{dw}{dq} + 5q^4$$

$$2w \cdot \frac{dw}{dq} \cdot q^3 - \cos(w^2) \cdot 2w \cdot \frac{dw}{dq} = 5q^4 - 3w^2q^2$$

$$\frac{dw}{dq}(2wq^3 - 2w\cos(w^2)) = 5q^4 - 3w^2q^2$$

$$\frac{dw}{dq} = \frac{5q^4 - 3w^2q^2}{2wq^3 - 2w\cos(w^2)}$$

2. (2 points) Compute  $f'(x)$  given that  $f(x) = \tan^{-1}(x^3)$ .

$$f'(x) = \frac{1}{1+(x^3)^2} \cdot (x^3)'$$

$$= \frac{1}{1+x^6} \cdot 3x^2$$

$$= \frac{3x^2}{1+x^6}$$

3. (3 points) Compute  $h'(t)$  given that  $h(t) = \ln(\tan(t^5 + 2t))$ .

$$\begin{aligned} h'(t) &= \frac{1}{\tan(t^5 + 2t)} \cdot (\tan(t^5 + 2t))' \\ &= \frac{1}{\tan(t^5 + 2t)} \cdot \sec^2(t^5 + 2t) \cdot (t^5 + 2t)' \\ &= \frac{1}{\tan(t^5 + 2t)} \cdot \sec^2(t^5 + 2t) \cdot (5t^4 + 2) \end{aligned}$$

4. (2 points) Compute  $\frac{dy}{dx}$  given that  $y = (\sec x)^{x^3}$ .

$$\ln(y) = \ln((\sec x)^{x^3})$$

$$\ln(y) = x^3 \cdot \ln(\sec x)$$

$$\frac{d}{dx}(\ln(y)) = \frac{d}{dx}(x^3 \cdot \ln(\sec x))$$

$$\frac{1}{y} \cdot \frac{dy}{dx} = \frac{d}{dx}(x^3) \cdot \ln(\sec x) + x^3 \cdot \frac{d}{dx}(\ln(\sec x))$$

$$\frac{1}{y} \cdot \frac{dy}{dx} = 3x^2 \cdot \ln(\sec x) + x^3 \cdot \frac{1}{\sec x} \cdot (\sec x)'$$

$$\frac{dy}{dx} = y \left( 3x^2 \ln(\sec x) + x^3 \cdot \frac{1}{\sec x} \cdot \sec x \tan x \right)$$

$$\frac{dy}{dx} = (\sec x)^{x^3} \left( 3x^2 \ln(\sec x) + x^3 \tan x \right)$$