1. (4 points) Find an equation of the line which is tangent to the curve \( y = x^3 - x^2 - 6x \) at its positive \( x \)-intercept.

\[
\text{set } y = 0 \\
0 = x^3 - x^2 - 6x \\
= x(x^2 - x - 6) \\
= x(x + 2)(x - 3)
\]

\( x \)-intercepts are at \( x = 0, x = -2, x = 3 \)

\( y' = 3x^2 - 2x - 6 \)

\( \text{Point: } (3, 0) \)

\( \text{Slope: } y'(3) = 3 \cdot 3^2 - 2 \cdot 3 - 6 = 15 \)

\( \text{Tangent line: } y - 0 = 15(x - 3) \)

or \( y = 15x - 45 \)

2. (2 points each) Using Leibniz notation (i.e., \( \frac{dy}{dx}, \frac{dp}{dt} \), etc.), find derivatives for each of the following functions. For part (a) simplify your answer.

(a) \( w = \left( \frac{\sqrt{x}}{x\sqrt{x}} \right)^{-6} \)

\[
= \left( \frac{x^{1/2}}{x \cdot x^{1/2}} \right)^{-6} = \left( \frac{x^{1/2}}{x^{3/2}} \right)^{-6} \\
= \left( x^{1/2 - 3/2} \right)^{-6} = \left( x^{-8/6} \right)^{-6} \\
= x^8
\]

so \( \frac{dw}{dx} = 8x^7 \)
(b) \( y = 5r^3e^r + \sin \left( \frac{\pi}{7} \right) \)

\[
\frac{dy}{dr} = \frac{d}{dr}(5r^3e^r) + (5r^3)\frac{d}{dr}(e^r) + 0
\]

\[
\frac{dy}{dr} = 15r^2e^r + 5r^3e^r
\]

Note: \( \sin \left( \frac{\pi}{7} \right) \) is a constant so its derivative is 0.

(c) \( q = \frac{8}{t^4 + 9} = 8 \left( t^4 + 9 \right)^{-1} \)

\[
\frac{dq}{dt} = 8\cdot -\left( t^4 + 9 \right)^{-2} \cdot 4t^3
\]

\[
\frac{dq}{dt} = -32t^3 \left( t^4 + 9 \right)^{-2}
\]

or use quotient rule

\[
\frac{dq}{dt} = \frac{\left( 8 \right)'}{\left( t^4 + 9 \right)'} - 8 \left( t^4 + 9 \right)'
\]

\[
\frac{dq}{dt} = \frac{0 - 8 \cdot 4t^3}{(t^4 + 9)^2}
\]

\[
= \frac{-32t^3}{(t^4 + 9)^2}
\]