Name solutions

- You have 15 minutes
- No calculators
- Show sufficient work

1. (4 points) Find an equation of the line which is tangent to the curve $y = x^3 - x^2 - 6x$ at its positive x-intercept.

set $y=0$

$$0 = x^3 - x^2 - 6x = x(x^2 - x - 6) = x(x+2)(x-3)$$

positive x-intercept

x-intercepts are at $x=0, x=-2, x=3$

$y' = 3x^2 - 2x - 6$

Point: $(3,0)$

slope: $y'(3) = 3\cdot 3^2 - 2\cdot 3 - 6 = 15$

Tangent line: $y - 0 = 15(x-3)$

or $y = 15x - 45$

2. (2 points each) Using Leibniz notation (i.e., $\frac{dy}{dx}$, $\frac{dP}{dt}$, etc.), find derivatives for each of the following functions. For part (a) simplify your answer.

(a) $w = \left(\frac{\sqrt[6]{x}}{x\sqrt{x}}\right)^{-6}$ $= \left(\frac{x^{1/6}}{x\cdot x^{1/2}}\right)^{-6} = \left(\frac{x^{1/6}}{x^{3/2}}\right)^{-6}$

$$= \left(x^{\frac{1}{6}-\frac{3}{2}}\right)^{-6} = \left(x^{-\frac{8}{6}}\right)^{-6}$$

$$= x^8$$

so $\frac{dw}{dx} = 8x^7$

(b) $y = 5r^3e^r + \sin\left(\frac{\pi}{7}\right)$

$$\frac{dy}{dr} = \frac{d}{dr}(5r^3)(e^r) + (5r^3)\frac{d}{dr}(e^r) + 0$$

$$\frac{dy}{dr} = 15r^2e^r + 5r^3e^r$$

note: $\sin\left(\frac{\pi}{7}\right)$ is a constant so its derivative is 0.

(c) $q = \frac{8}{t^4+9}$ $= 8(t^4+9)^{-1}$

$$\frac{dq}{dt} = 8 \cdot -(t^4+9)^{-2} \cdot 4t^3$$

$$\frac{dq}{dt} = -32t^3(t^4+9)^{-2}$$

or use quotient rule

$$\frac{dq}{dt} = \frac{(8)'(t^4+9) - 8(t^4+9)'}{(t^4+9)^2}$$

$$= \frac{0 - 8 \cdot 4t^3}{(t^4+9)^2}$$

$$= \frac{-32t^3}{(t^4+9)^2}$$