Name Solutions

- You have 15 minutes
- No calculators
- Show sufficient work

1. (4 points) A bacterial culture starts with 200 bacteria and triples in size every 2 hours.

   (a) Find a formula for the number of bacteria as a function of the number of hours since its population was 200.

t	P
0	200
2	600
4	1800
6	5400
⋮	⋮

$P = C \cdot a^t$

at $(t, P) = (0, 200)$ this gives

$200 = C \cdot a^0 \Rightarrow C = 200$

$P = 200 \cdot a^t$

at $(t, P) = (2, 600)$ this gives

$600 = 200 \cdot a^2 \Rightarrow a = \pm\sqrt{3}$

using $a = \sqrt{3}$ we obtain

$P = 200 \cdot (\sqrt{3})^t = 200 \cdot 3^{t/2}$

   (b) At what time is the population equal to 1000 ?

$1000 = 200 \cdot 3^{t/2}$

$5 = 3^{t/2}$

$\ln(5) = \ln(3^{t/2})$

$\ln(5) = \frac{t}{2}\ln(3)$

$t = \frac{2\ln(5)}{\ln(3)}$ hours

2. (3 points) Determine the exact value for each solution to the equation below.

$$\ln(4-x)+\ln(4+x)=0$$

$\ln((4-x)(4+x))=0$

$\ln(16-x^2)=0$

$e^0=16-x^2$

$1=16-x^2$

$x^2=15$

$x=\pm\sqrt{15}$

note that both $x=-\sqrt{15}$ and $x=\sqrt{15}$ are in the domain of $\ln(4-x)+\ln(4+x)$.

3. (3 points) Given that $g(x)=\dfrac{3+2x}{5-4x}$, find a formula for $g^{-1}(x)$.

Let $y=g^{-1}(x)$

Then $g(y)=x$

$\dfrac{3+2y}{5-4y}=x$

$3+2y=x(5-4y)$

$3+2y=5x-4xy$

$4xy+2y=5x-3$

$(4x+2)y=5x-3$

$y=\dfrac{5x-3}{4x+2}$

$g^{-1}(x)=\dfrac{5x-3}{4x+2}$