Math 220 **Quiz 12 (take-home)** **Fall 2012**

Name SOLUTIONS

(circle your TA discussion section)

- ▷ **AD1**, TR 11:00-12:50, Hannah Kolb Spinoza
- ▷ **AD3**, TR 1:00-2:50, Michael Santana
- ▷ **ADB**, TR 9:00-9:50, Ziying Pan
- ▷ **ADD**, TR 11:00-11:50, Lisa Hickok
- ▷ **ADF**, TR 1:00-1:50, Jian Liang
- ▷ **ADH**, TR 3:00-3:50, Lechao Xiao
- ▷ **ADJ**, TR 9:00-9:50, Meghan Galiardi
- ▷ **ADL**, TR 11:00-11:50, Andrew McConvey
- ▷ **ADN**, TR 1:00-1:50, Benjamin Fulan
- ▷ **ADP**, TR 3:00-3:50, Hongfei Tian
- ▷ **ADR**, TR 9:00-9:50, Noah Chartoff
- ▷ **ADT**, TR 2:00-2:50, Anna Weigandt
- ▷ **AD2**, TR 9:00-10:50, Ki Yeun Kim
- ▷ **ADA**, TR 8:00-8:50, Ziying Pan
- ▷ **ADC**, TR 10:00-10:50, Lisa Hickok
- ▷ **ADE**, TR 12:00-12:50, Andrew McConvey
- ▷ **ADG**, TR 2:00-2:50, Derrek Yager
- ▷ **ADI**, TR 4:00-4:50, Lechao Xiao
- ▷ **ADK**, TR 10:00-10:50, Meghan Galiardi
- ▷ **ADM**, TR 12:00-12:50, Benjamin Fulan
- ▷ **ADO**, TR 2:00-2:50, Jian Liang
- ▷ **ADQ**, TR 4:00-4:50, Hongfei Tian
- ▷ **ADS**, TR 12:00-12:50, Derrek Yager
- ▷ **ADU**, TR 3:00-3:50, Anna Weigandt

- You may work with other MATH 220 students. However each student should write up solutions separately and independently – nobody should copy someone else's work.
- You may use your notes or the textbook.
- **The quiz should be submitted to Mr. Murphy at the beginning of your official lecture period on Friday, November 30th.**
- There is a higher expectation for the quality of your work on a take-home quiz. Everything should be written logically and legibly with sufficient work to justify each answer. Blank copies of the quiz are available on the course home page.
- Be sure that the pages are nicely stapled – do not just fold the corners.
- **Note to TAs and Tutors – you should not help students with these specific problems or go over solutions until after 5pm Friday.**

1. (5 points) Determine an appropriate linear approximation of the function $f(x) = \sqrt[3]{x}$ and use it to approximate $\sqrt[3]{999.25}$. Write your answer in decimal form. No technology is allowed on this problem.

Since we know $\sqrt[3]{1000} = 10$ and 999.25 is close to 1000,

we will use the tangent line to $f(x) = \sqrt[3]{x}$ at 1000 to approximate our result.

$f(x) = \sqrt[3]{x} \Rightarrow f(1000) = 10 \Rightarrow$ point is $(1000, 10)$

$f'(x) = \frac{1}{3}x^{-2/3} \Rightarrow f'(1000) = \frac{1}{3} \cdot 1000^{-2/3} = \frac{1}{300}$

$\Rightarrow$ slope $= \frac{1}{300}$

equation for tangent line is $y - 10 = \frac{1}{300}(x - 1000)$

$y = 10 + \frac{1}{300}(x - 1000)$

Thus $\sqrt[3]{x} \approx 10 + \frac{1}{300}(x - 1000)$ for x near 1000

$\sqrt[3]{999.25} \approx 10 + \frac{1}{300}(999.25 - 1000)$

$\sqrt[3]{999.25} \approx 10 - \frac{1}{400}$

$\sqrt[3]{999.25} \approx 9.9975$

2. (5 points) The graphs of $f(x) = 2x^3$ and $g(x) = 5x + 4$ have one intersection point. Determine the x-value for this intersection point using Newton's Method with an initial estimate of $x_1 = 4$. You should use this method 3 times in order to obtain estimates x_2, x_3 and x_4. You are only allowed to use technology for basic arithmetic. Use at least 5 decimal places in each estimate.

$2x^3 = 5x + 4 \Rightarrow 2x^3 - 5x - 4 = 0$

Let $f(x) = 2x^3 - 5x - 4$

$f'(x) = 6x^2 - 5$

Apply Newton's Method to $f(x)$ with an initial estimate of $x_1 = 4$ for a root of f.

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)} = x_n - \frac{2x_n^3 - 5x_n - 4}{6x_n^2 - 5}$$

$x_1 = 4$

$$x_2 = 4 - \frac{2(4)^3 - 5(4) - 4}{6(4)^2 - 5} \approx 2.857142857$$

$$x_3 \approx 2.857142857 - \frac{2(2.857142857)^3 - 5(2.857142857) - 4}{6(2.857142857)^2 - 5}$$

$x_3 \approx 2.212263838$

$$x_4 \approx 2.212263838 - \frac{2(2.212263838)^3 - 5(2.212263838) - 4}{6(2.212263838)^2 - 5}$$

$x_4 \approx 1.941674814$