Math 220

Quiz 12 (take-home)

Fall 2012

(circle your TA discussion section)

▷ AD1, TR 11:00-12:50, Hannah Kolb Spinoza
▷ AD2, TR 9:00-10:50, Ki Yeun Kim
▷ AD3, TR 1:00-2:50, Michael Santana
▷ ADA, TR 8:00-8:50, Ziyiing Pan
▷ ADB, TR 9:00-9:50, Ziying Pan
▷ ADC, TR 10:00-10:50, Lisa Hickok
▷ ADD, TR 11:00-11:50, Lisa Hickok
▷ ADE, TR 12:00-12:50, Andrew McConvey
▷ ADF, TR 1:00-1:50, Jian Liang
▷ ADG, TR 2:00-2:50, Derrek Yager
▷ ADH, TR 3:00-3:50, Lechao Xiao
▷ ADI, TR 4:00-4:50, Lechao Xiao
▷ ADJ, TR 9:00-9:50, Meghan Galiardi
▷ ADK, TR 10:00-10:50, Meghan Galiardi
▷ ADL, TR 11:00-11:50, Andrew McConvey
▷ ADM, TR 12:00-12:50, Benjamin Fulan
▷ ADN, TR 1:00-1:50, Benjamin Fulan
▷ ADO, TR 2:00-2:50, Jian Liang
▷ ADP, TR 3:00-3:50, Hongfei Tian
▷ ADQ, TR 4:00-4:50, Hongfei Tian
▷ ADR, TR 9:00-9:50, Noah Chartoff
▷ ADS, TR 12:00-12:50, Derrek Yager
▷ ADT, TR 2:00-2:50, Anna Weigandt
▷ ADU, TR 3:00-3:50, Anna Weigandt

- You may work with other MATH 220 students. However each student should write up solutions separately and independently – nobody should copy someone else’s work.

- You may use your notes or the textbook.

- The quiz should be submitted to Mr. Murphy at the beginning of your official lecture period on Friday, November 30th.

- There is a higher expectation for the quality of your work on a take-home quiz. Everything should be written logically and legibly with sufficient work to justify each answer. Blank copies of the quiz are available on the course home page.

- Be sure that the pages are nicely stapled – do not just fold the corners.

- Note to TAs and Tutors – you should not help students with these specific problems or go over solutions until after 5pm Friday.
1. (5 points) Determine an appropriate linear approximation of the function \( f(x) = \sqrt[3]{x} \) and use it to approximate \( \sqrt[3]{999.25} \). Write your answer in decimal form. No technology is allowed on this problem.

Since we know \( \sqrt[3]{1000} = 10 \) and 999.25 is close to 1000, we will use the tangent line to \( f(x) = \sqrt[3]{x} \) at 1000 to approximate our result.

\[
\begin{align*}
&f(x) = \sqrt[3]{x} \\&f(1000) = 10 \\&\text{point is } (1000, 10) \\&f'(x) = \frac{1}{3}x^{-\frac{2}{3}} \\&f'(1000) = \frac{1}{3} \cdot 1000^{-\frac{2}{3}} = \frac{1}{300} \\&\text{slope } = \frac{1}{300}
\end{align*}
\]

Equation for tangent line is
\[
y - 10 = \frac{1}{300} (x - 1000)
\]

Thus, \( \sqrt[3]{x} \approx 10 + \frac{1}{300} (x - 1000) \) for \( x \) near 1000.

\[
\begin{align*}
&\sqrt[3]{999.25} \approx 10 + \frac{1}{300} (999.25 - 1000) \\
&\sqrt[3]{999.25} \approx 10 - \frac{1}{400} \\
&\sqrt[3]{999.25} \approx 9.9975
\end{align*}
\]
2. (5 points) The graphs of $f(x) = 2x^3$ and $g(x) = 5x + 4$ have one intersection point. Determine the $x$-value for this intersection point using Newton's Method with an initial estimate of $x_1 = 4$. You should use this method 3 times in order to obtain estimates $x_2$, $x_3$ and $x_4$. You are only allowed to use technology for basic arithmetic. Use at least 5 decimal places in each estimate.

$$2x^3 - 5x - 4 = 0$$

Let $f(x) = 2x^3 - 5x - 4$

$$f'(x) = 6x^2 - 5$$

Apply Newton's Method to $f(x)$ with an initial estimate of $x_1 = 4$ for a root of $f$.

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)} = x_n - \frac{2x_n^3 - 5x_n - 4}{6x_n^2 - 5}$$

$x_1 = 4$

$x_2 = 4 - \frac{2(4)^3 - 5(4) - 4}{6(4)^2 - 5} = 2.857142857$

$x_3 \approx 2.857142857 - \frac{2(2.857142857)^3 - 5(2.857142857) - 4}{6(2.857142857)^2 - 5} = 2.212263838$

$x_4 \approx 2.212263838 - \frac{2(2.212263838)^3 - 5(2.212263838) - 4}{6(2.212263838)^2 - 5} = 1.941674814$