Math 220 (section AD?) Quiz 11 Fall 2012

Name Solutions

- You have 15 minutes
- No calculators
- Show sufficient work

1. (4 points) Determine the average value of the function $f(x) = \dfrac{10x}{e^{x^2}}$ on the interval $[1, 3]$.

$$f_{ave} = \frac{1}{3-1}\int_1^3 \frac{10x}{e^{x^2}}\,dx$$

$$= \frac{1}{2}\int_1^3 10x e^{-x^2}\,dx$$

$$= \frac{1}{2}\int_{-1}^{-9} -5e^u\,du$$

$$\left(\begin{array}{l} u = -x^2 \\ du = -2x\,dx \\ -5\,du = 10x\,dx \\ x = 1 \Rightarrow u = -1^2 = -1 \\ x = 3 \Rightarrow u = -3^2 = -9 \end{array}\right)$$

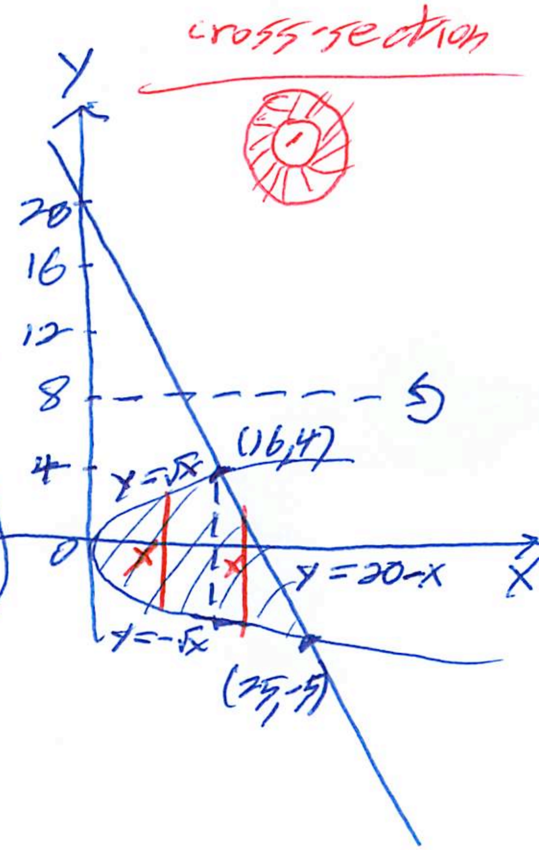
$$= \frac{-5}{2}e^u\Big|_{-1}^{-9}$$

$$= \frac{-5}{2}e^{-9} - \frac{-5}{2}e^{-1}$$

$$= \frac{-5}{2e^9} + \frac{5}{2e}$$

2. (3 points each) Let $\mathbf{R}$ be the finite region bounded by $x = y^2$ and $x = 20 - y$. In the following manner set up, but do not evaluate, definite integrals which represent the volume of the solid obtained when $\mathbf{R}$ is revolved around the horizontal line $y = 8$.

(a) Integrate with respect to x.



intersection $y^2 = 20 - y$

$y^2 + y - 20 = 0$

$(y+5)(y-4) = 0$

$y = -5, \; x = 25$; $y = 4, \; x = 16$

$$V = \int_0^{16} \left(\pi(8-(-\sqrt{x}))^2 - \pi(8-\sqrt{x})^2\right)dx + \int_{16}^{25} \left(\pi(8-(-\sqrt{x}))^2 - \pi(8-(20-x))^2\right)dx$$

(b) Integrate with respect to y. (The integrands in parts (a) and (b) should be different.)

$$V = \int_{-5}^{4} 2\pi r h \, dy$$

$$V = \int_{-5}^{4} 2\pi (8-y)(20-y-y^2)\,dy$$