Math 220 (section AD?) Quiz 10 Fall 2012

Name Solutions

- You have 15 minutes
- No calculators
- Show sufficient work

1. (3 points) Suppose $w(x) = \int_{10}^{x^2} (t-4)(t+1)^6\,dt$. Determine all intervals upon which the function $w(x)$ is increasing.

$w = \int_{10}^{x^2} (t-4)(t+1)^6\,dt$

Let $u = x^2$ so $w = \int_{10}^{u} (t-4)(t+1)^6\,dt$

$\frac{du}{dx} = 2x$ and $\frac{dw}{du} = (u-4)(u+1)^6$ by F.T.C. (part I)

$w'(x) = \frac{dw}{dx} = \frac{dw}{du}\cdot\frac{du}{dx} = (u-4)(u+1)^6 \cdot 2x$

we can actually get this directly

$\boxed{w'(x) = (x^2-4)(x^2+1)^6 \cdot 2x}$

$= 2x(x-2)(x+2)(x^2+1)^6$

values of $w'(x)$

$$\begin{array}{ccccccccc} -- & 0 & ++ & 0 & -- & 0 & ++ & \\ & -2 & & 0 & & 2 & & \to x \end{array}$$

w is increasing on $[-2, 0]$ and $[2, \infty)$

2. (2 points) Precisely state *The Mean Value Theorem.*

Let f be a function that satisfies the following hypotheses:

① f is continuous on the closed interval $[a,b]$.

② f is differentiable on the open interval (a,b).

Then there is a number c in (a,b) such that

$$f'(c) = \frac{f(b)-f(a)}{b-a}$$

3. (3 points) Evaluate the following definite integral.

$$\int_0^1 \frac{6x+3}{x^2+x+5}\,dx$$

Let $u = x^2+x+5$, $x=0 \Rightarrow u = 0^2+0+5=5$, $x=1 \Rightarrow u = 1^2+1+5=7$

$\frac{du}{dx} = 2x+1 \Rightarrow du = (2x+1)dx \Rightarrow 3du = (6x+3)dx$

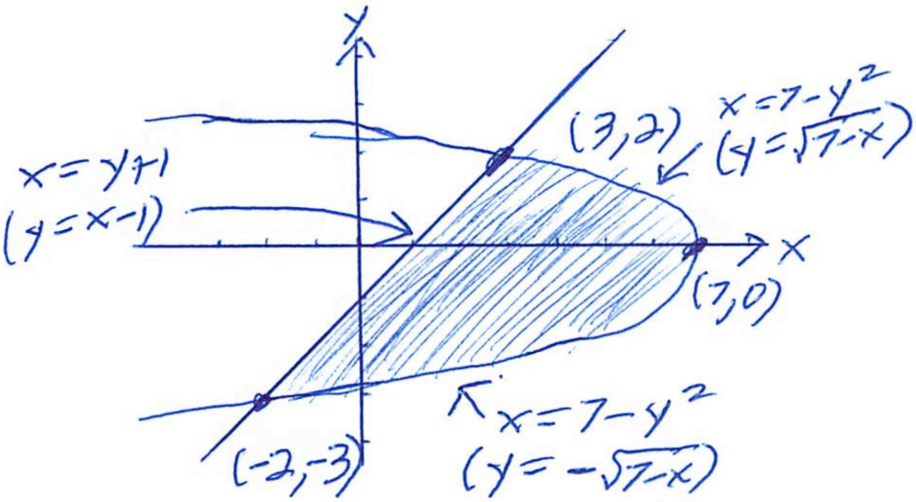
$$\int_0^1 \frac{6x+3}{x^2+x+5}dx = \int_5^7 \frac{3du}{u} = 3\int_5^7 \frac{1}{u}\,du$$

$$= \Big[3\ln|u|\Big]_5^7$$

$$= 3\ln(7) - 3\ln(5)$$

or $3\ln\left(\frac{7}{5}\right)$

4. (2 points) Set up, but do not evaluate, one or more integrals which represent the area of the finite region bounded by the curves $y = x-1$ and $x = 7-y^2$.



intersection points

$x=y+1$ & $x=7-y^2$

so $y+1 = 7-y^2$

$y^2+y-6=0$

$(y+3)(y-2)=0$

$y=-3,\ y=2$

$y=-3 \Rightarrow x=-2$

$y=2 \Rightarrow x=3$

$$\text{area} = \int_{-3}^{2}\left((7-y^2)-(y+1)\right)dy$$

easier to set up and integrate with respect to y

or

$$\text{area} = \int_{-2}^{3}\left((x-1)-(-\sqrt{7-x})\right)dx + \int_3^7\left(\sqrt{7-x}-(-\sqrt{7-x})\right)dx$$