1. (3 points) Suppose \( w(x) = \int_{10}^{x^2} (t - 4)(t + 1)^6 \, dt \). Determine all intervals upon which the function \( w(x) \) is increasing.

\[
\begin{align*}
w &= \int_{10}^{x^2} (t - 4)(t + 1)^6 \, dt \\
\text{Let } u &= x^2 \text{ so } w &= \int_{10}^{u} (t - 4)(t + 1)^6 \, dt \\
\frac{du}{dx} &= 2x \quad \text{and} \quad \frac{dw}{du} = (u - 4)(u + 1)^6 \quad \text{by FTC, (part 1)} \\
w'(x) &= \frac{dw}{du} \cdot \frac{du}{dx} = (u - 4)(u + 1)^6 \cdot 2x \\
w'(x) &= (x^2 - 4)(x^2 + 1)^6 \cdot 2x \\
10 \leq x^2 \leq 10 \implies x \in [-\sqrt{10}, \sqrt{10}] \\
\end{align*}
\]

Therefore, \( w \) is increasing on \([-\sqrt{10}, 0]\) and \([0, \sqrt{10}]\).

2. (2 points) Precisely state The Mean Value Theorem.

Let \( f \) be a function that satisfies the following hypotheses:
① \( f \) is continuous on the closed interval \([a, b]\).
② \( f \) is differentiable on the open interval \((a, b)\).

Then there is a number \( c \) in \((a, b)\) such that
\[
f'(c) = \frac{f(b) - f(a)}{b - a}
\]
3. (3 points) Evaluate the following definite integral.

\[
\int_0^1 \frac{6x + 3}{x^2 + x + 5} \, dx
\]

Let \( u = x^2 + x + 5 \), \( \Rightarrow x = 0 \Rightarrow u = 0^2 + 0 + 5 = 5 \), \( x = 1 \Rightarrow u = 1^2 + 5 = 6 \).

\[
\frac{du}{dx} = 2x + 1 \Rightarrow 
\int \frac{6x + 3}{x^2 + x + 5} \, dx = \int \frac{3}{u} \, du = \frac{3}{5} \ln(u)
\]

\[
\left[ \frac{3}{5} \ln(u) \right]_5^7 = 3 \ln(7) - 3 \ln(5) = 3 \ln \left( \frac{7}{5} \right)
\]

or \( 3 \ln \left( \frac{7}{5} \right) \)

4. (2 points) Set up, but do not evaluate, one or more integrals which represent the area of the finite region bounded by the curves \( y = x - 1 \) and \( x = 7 - y^2 \).

**Intersection Points**

\[
x = y + 1 \quad \text{&} \quad x = 7 - y^2
\]

So \( y + 1 = 7 - y^2 \)

\( y^2 + y - 6 = 0 \)

\( (y + 3)(y - 2) = 0 \)

\( y = -3 \), \( y = 2 \)

\( y = -3 \Rightarrow x = -2 \)

\( y = 2 \Rightarrow x = 3 \)

**Area**

**Easier to set up and integrate with respect to** \( y \)

\[
\text{area} = \int_{-3}^{2} \left( 7-y^2 - (y+1) \right) \, dy
\]

\[
\text{or}
\int_{-2}^{3} \left( x - 1 \right) \, dx + \int_{3}^{2} \left( 7 - x - (-\sqrt{7-x}) \right) \, dx
\]