

Name SOLUTIONS

- No calculators allowed.
- Show sufficient work to justify each answer.
- You have 15 minutes for this quiz.

1. (2 points) Find any antiderivative of $f(x) = \frac{x^3 - 16x^4 \sec^2 x + 8}{8x^4}$.

$$f(x) = \frac{x^3}{8x^4} - \frac{16x^4 \sec^2 x}{8x^4} + \frac{8}{8x^4}$$

$$f(x) = \frac{1}{8} \cdot \frac{1}{x} - 2 \sec^2 x + x^{-4}$$

$$F(x) = \frac{1}{8} \ln|x| - 2 \tan x - \frac{1}{3} x^{-3}$$

is an antiderivative of $f(x)$

2. (3 points) Find $g(t)$ given that $g''(t) = 24t^2 + 4e^t$, $g'(0) = 7$ and $g(0) = 10$.

$$g'(t) = 8t^3 + 4e^t + C$$

$$g'(0) = 7 \Rightarrow 7 = 8 \cdot 0^3 + 4e^0 + C$$

$$7 = 4 + C$$

$$C = 3$$

$$g'(t) = 8t^3 + 4e^t + 3$$

$$g(t) = 2t^4 + 4e^t + 3t + D$$

$$g(0) = 10 \Rightarrow 10 = 2 \cdot 0^4 + 4e^0 + 3 \cdot 0 + D$$

$$10 = 4 + D$$

$$D = 6$$

$$g(t) = 2t^4 + 4e^t + 3t + 6$$

3. (3 points) The acceleration due to gravity near the surface of some planet is -8 m/s^2 . An object is shot upward from the surface of this planet and 12 seconds later it has fallen back to the surface. What is the initial velocity of this object?

acc., $s''(t) = -8$

vel., $s'(t) = -8t + C$

pos., $s(t) = -4t^2 + Ct + D$

$$s(0) = 0 \Rightarrow D = 0 \Rightarrow s(t) = -4t^2 + Ct$$

$$s(12) = 0 \Rightarrow 0 = -4(12)^2 + C(12)$$

$$\Rightarrow C = \frac{4(12)^2}{12} = 48$$

Thus $s'(t) = -8t + 48$

The initial velocity is

$$s'(0) = -8 \cdot 0 + 48 = 48 \text{ m/s}$$

4. (2 points) Coal gas is produced at a gasworks. Pollutants in the gas are removed by scrubbers, which become less and less efficient as time goes on. The following measurements, made at the start of every other month, show the rate (in tons per month) at which pollutants are escaping in the gas over the course of one year.

Time t (months)	0	2	4	6	8	10	12
Rate pollutants escape (tons/month)	2.5	8.0	12.5	15.0	16.5	17.5	18.0

Use a left Riemann sum with $\Delta t = 6$ to approximate the total amount of pollutants that escape over the course of the year. Simplify your answer and include proper units.

amount of escaped pollutants

$$\begin{aligned} &\approx \left(2.5 \frac{\text{tons}}{\text{month}}\right) (6 \text{ months}) \\ &\quad + \left(15.0 \frac{\text{tons}}{\text{month}}\right) (6 \text{ months}) \\ &= (15 + 90) \text{ tons} \\ &= 105 \text{ tons} \end{aligned}$$