SOLUTIONS

- You may work with other students in this class. However each student should write up solutions separately and independently – nobody should copy someone else’s work.

- You may use your notes or the textbook.

- Computers are not allowed on any problem. You may use a calculator only for basic arithmetic.

- You must show sufficient work to justify each answer.

- The quiz should be turned in to your TA at the beginning of your discussion section meeting on Thursday, October 13th.

- Be sure that the pages are nicely stapled – do not just fold the corners.

- Note to TAs and Tutors – you should not help students with these specific problems or go over solutions until after 4pm Thursday.

1. (3 points) Determine the x-value for each inflection point on the graph of the following function.

\[ f(x) = 3x^5 - 5x^4 - 80x^3 + 360x^2 + 1000x + 850 \]

\[ f'(x) = 15x^4 - 20x^3 - 240x^2 + 720x + 1000 \]

\[ f''(x) = 60x^3 - 60x^2 - 480x + 720 \]

\[ = 60(x^3 - x^2 - 8x + 12) \]

\[ = 60(x - 2)^2(x + 3) \]

\[ \text{values of } f''(x) \]

\[ -3 \quad 0 \quad 1 \quad 2 \quad 1 \]

\[ f \text{ switches from concave down to concave up at } x = -3 \text{ so there is an inflection point at } x = -3. \text{ [note: } f(-3) = (-3, f(-3)) = (-3, 2116)] \]
2. (3 points) Suppose the function $f$ has first derivative as shown below.

$$f'(x) = e^{2x} (x^2 + 25) (x - 3)^2 (x^2 - 64) (2x - 1)$$

List each interval upon which the function $f$ is decreasing.

\[ f' < 0 \Rightarrow f \text{ is decreasing on intervals } (-\infty, -8), (\frac{3}{2}, 3) \text{ and } (3, 8) \]

Actually from the definition on page 19, $f$ is decreasing on intervals $(-\infty, -8]$ and $[\frac{3}{2}, 8]$.

3. (4 points) A farmer wishes to enclose a rectangular pen with area 100 square feet next to a road. The fence along the road is to be reinforced and costs $34 per foot. Fencing that costs $16 per foot can be used for the other three sides. What dimensions for the pen will minimize the cost to the farmer? What is that minimum cost?

\[
\text{area} = 100 \text{ ft}^2 \Rightarrow xy = 100 \Rightarrow y = \frac{100}{x}
\]

Total cost:

\[
C = 34x + 16(x + 2y)
\]

\[
C = 50x + 32y
\]

\[
C = 50x + 32\left(\frac{100}{x}\right) = 50x + \frac{3200}{x}
\]

\[
C' = 50 - \frac{3200}{x^2}
\]

Note: $x$ must be positive since it is a length of fencing.

Minimum cost at $x = 8$ ft:

\[
y = \frac{100}{8} = 12.5 \text{ ft}
\]

Dimensions: $8 \text{ ft} \times 12.5 \text{ ft}$

Minimum cost is $C = 50(8) + 32(12.5)$

Min cost: $\$800$