Math 220 Quiz 8 (take-home) Fall 2011

Name SOLUTIONS

- You may work with other students in this class. However each student should write up solutions separately and independently – nobody should copy someone else's work.
- You may use your notes or the textbook.
- Computers are not allowed on any problem. You may use a calculator only for basic arithmetic.
- You must show sufficient work to justify each answer.
- The quiz should be turned in to your TA at the beginning of your discussion section meeting on Thursday, October 13th.
- Be sure that the pages are nicely stapled – do not just fold the corners.
- **Note to TAs and Tutors – you should not help students with these specific problems or go over solutions until after 4pm Thursday.**

1. (3 points) Determine the x-value for each inflection point on the graph of the following function.

$$f(x) = 3x^5 - 5x^4 - 80x^3 + 360x^2 + 1000x + 850$$

$$f'(x) = 15x^4 - 20x^3 - 240x^2 + 720x + 1000$$

$$f''(x) = 60x^3 - 60x^2 - 480x + 720$$

$$= 60(x^3 - x^2 - 8x + 12)$$

$$= 60(x-2)^2(x+3)$$

values of $f''(x)$

$- - -$ 0 $+ + +$ 0 $+ + +$

x: -3, 2

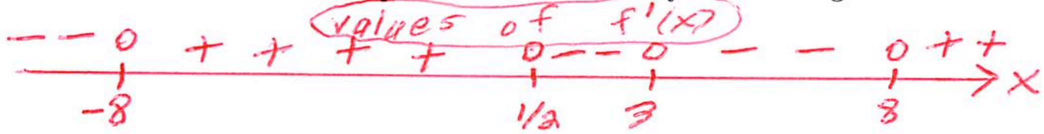
f switches from concave down to concave up at $x=-3$ so there is an inflection point at $x=-3$. [note: $(-3, f(-3)) = (-3, 2116)$]

2. (3 points) Suppose the function f has first derivative as shown below.

$$f'(x) = e^{2x}\left(x^2+25\right)(x-3)^2\left(x^2-64\right)(2x-1)$$

$$= e^{2x}(x^2+25)(x-3)^2(x-8)(x+8)(2x-1)$$

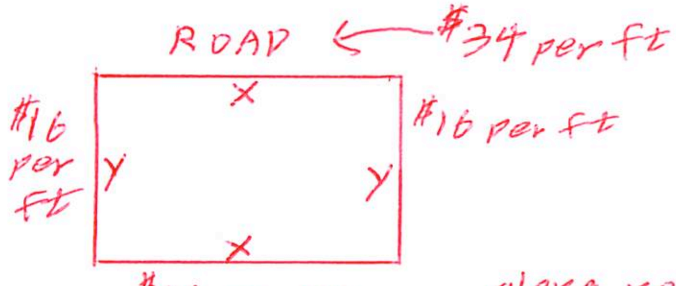
List each interval upon which the function f is decreasing.



$f' < 0 \Rightarrow f$ is decreasing on intervals $(-\infty,-8)$, $(\frac{1}{2},3)$ and $(3,8)$.

Actually from the definition on page 19, f is decreasing on intervals $(-\infty,-8]$ and $[\frac{1}{2},8]$

3. (4 points) A farmer wishes to enclose a rectangular pen with area 100 square feet next to a road. The fence along the road is to be reinforced and costs \$34 per foot. Fencing that costs \$16 per foot can be used for the other three sides. What dimensions for the pen will minimize the cost to the farmer? What is that minimum cost?



area $= 100\ \text{ft}^2 \Rightarrow xy = 100$

$\Rightarrow y = 100/x$

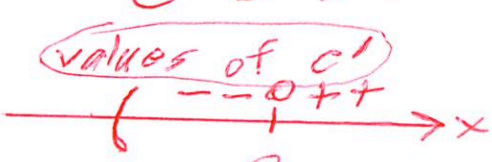
$$\text{Total cost} = \underbrace{(\text{cost per ft})(\#\text{ft})}_{\text{along road}} + \underbrace{(\text{cost per ft})(\#\text{ft})}_{\text{other 3 sides}}$$

$$C = 34x + 16(x+2y)$$

$$C = 50x + 32y$$

$$C = 50x + 32\left(\frac{100}{x}\right) = 50x + 3200x^{-1}$$

$$C' = 50 - 3200x^{-2} = \frac{50(x-8)(x+8)}{x^2}$$



note: x must be positive since it is a length of fencing

minimum cost at $x = 8$ ft,

$y = \frac{100}{8} = 12.5$ ft

Dimensions: 8 ft × 12.5 ft

min cost is $C = 50(8) + 32(12.5)$

min cost = \$800