Math 220 Quiz 7 (take-home) Fall 2011

Name SOLUTIONS

- You may work with other students in this class. However each student should write up solutions separately and independently – nobody should copy someone else's work.
- You may use your notes or the textbook.
- Computers are not allowed on any problem. You may use a calculator only for basic arithmetic.
- You must show sufficient work to justify each answer.
- The quiz should be turned in to your TA at the beginning of your discussion section meeting on Tuesday, October 11th.
- Be sure that the pages are nicely stapled – do not just fold the corners.
- **Note to TAs and Tutors – you should not help students with these specific problems or go over solutions until after 4pm Tuesday.**

1. (3 points) Suppose y_1 is a function of x for which $\frac{dy_1}{dx} = 3y_1$. Suppose y_2 is a function of x for which $\frac{dy_2}{dx} = 8x + 5$. If the graphs of y_1 and y_2 have the same y-intercept and they intersect at $x = 2$, then determine the value of the y-intercept.

$$\frac{dy_1}{dx} = 3y_1 \Rightarrow y_1 = c_1 e^{3x}$$

$$\frac{dy_2}{dx} = 8x+5 \Rightarrow y_2 = 4x^2+5x+c_2$$

same y-intercept $\Rightarrow y_1(0) = y_2(0)$

$$c_1 e^{3\cdot 0} = 4\cdot 0^2 + 5\cdot 0 + c_2$$

$$c_1 = c_2$$

Let $c = c_1 = c_2$

so $y_1 = ce^{3x}$ & $y_2 = 4x^2+5x+c$

intersect at $x=2$ $\Rightarrow y_1(2) = y_2(2)$

$$ce^{3\cdot 2} = 4\cdot 2^2 + 5\cdot 2 + c$$

$$ce^6 = 26 + c$$

$$ce^6 - c = 26$$

$$c(e^6-1) = 26$$

$$c = \frac{26}{e^6-1}$$

so $y_1 = \frac{26}{e^6-1} e^{3x}$ & $y_2 = 4x^2+5x+\frac{26}{e^6-1}$

The y-intercept is $\frac{26}{e^6-1}$

2. (4 points) A bullet is fired upwards from the ground at an initial velocity of 1200 feet per second. The height s (in feet) of the bullet above the ground after t seconds is $s = 1200t - 6t^2$ on Mars and $s = 1200t - 16t^2$ on Earth. How much higher will the bullet travel on Mars than on Earth?

Mars

height $s = 1200t - 6t^2$

velocity $s' = 1200 - 12t$

$= 12(100 - t)$

values of s'

+++ 0 ---

$\to t$

100

At $t = 100$ sec, the bullet reaches its maximum height of

$s(100) = 1200 \cdot 100 - 6 \cdot 100^2$

$= 60000$ ft

Earth

height $s = 1200t - 16t^2$

velocity $s' = 1200 - 32t$

$= 16(75 - 2t)$

values of s'

+++ 0 ---

$\to t$

$\frac{75}{2}$

At $t = \frac{75}{2}$ sec, the bullet reaches its maximum height of

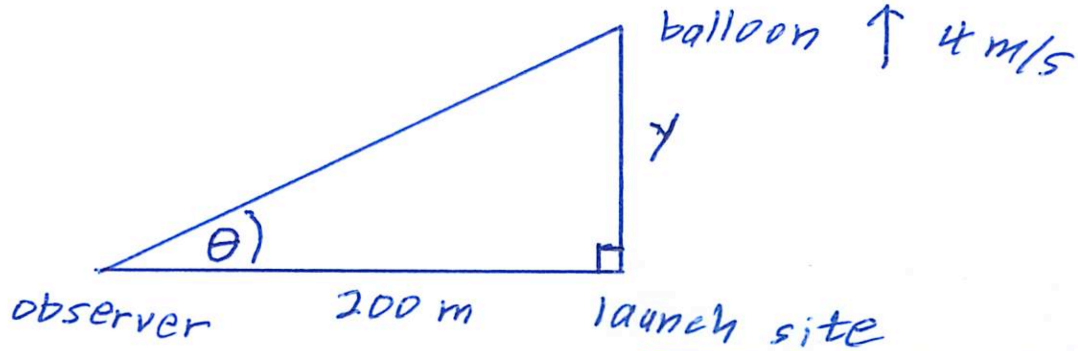
$s(\frac{75}{2}) = 1200 \cdot \frac{75}{2} - 16 \cdot (\frac{75}{2})^2$

$= 22500$ ft

Since $60000 - 22500 = 37500$,

the bullet will travel 37500 ft higher on Mars than on Earth.

3. (3 points) An observer stands 200 meters from the launch site of a hot-air balloon. The balloon rises vertically at a constant rate of 4 meters per second. How fast is the angle of elevation of the balloon increasing 30 seconds after the launch?



Given: $\frac{dy}{dt} = 4$ m/s

Want: $\left.\frac{d\theta}{dt}\right|_{t=30\,s}$

$$\tan\theta = \frac{y}{200}$$

$$\frac{d}{dt}(\tan\theta) = \frac{d}{dt}\left(\frac{y}{200}\right)$$

$$\sec^2\theta \cdot \frac{d\theta}{dt} = \frac{1}{200} \cdot \frac{dy}{dt}$$

why rad/s and not deg/s? Our short-cut derivative rules for trig functions are only valid in radians since the proofs used $\lim_{h \to 0} \frac{\sin(h)}{h} = 1$ which is only valid in radians

At $t = 30$ s, $y = (4 \text{ m/s}) \cdot (30 \text{ s}) = 120$ m

so that $\tan\theta = \frac{120}{200} = \frac{3}{5}$ and

$$\sec^2\theta = \tan^2\theta + 1 = \left(\frac{3}{5}\right)^2 + 1 = \frac{34}{25}$$

From $\sec^2\theta \cdot \frac{d\theta}{dt} = \frac{1}{200} \cdot \frac{dy}{dt}$ we obtain

$$\left(\frac{34}{25}\right) \cdot \frac{d\theta}{dt} = \frac{1}{200} \cdot (4)$$

$$\boxed{\frac{d\theta}{dt} = \frac{4}{200} \cdot \left(\frac{25}{34}\right) = \frac{1}{68} \approx 0.0147 \ \frac{\text{radians}}{\text{sec}}}$$