

Name SOLUTIONS

- No calculators allowed.
- Show sufficient work to justify each answer.
- You have 15 minutes for this quiz.

1. (4 points) Find the slope of the line tangent to the graph of the ellipse

$$\frac{x^2}{25} + \frac{y^2}{3} = 1$$

at the point $(\frac{5}{2}, \frac{3}{2})$.

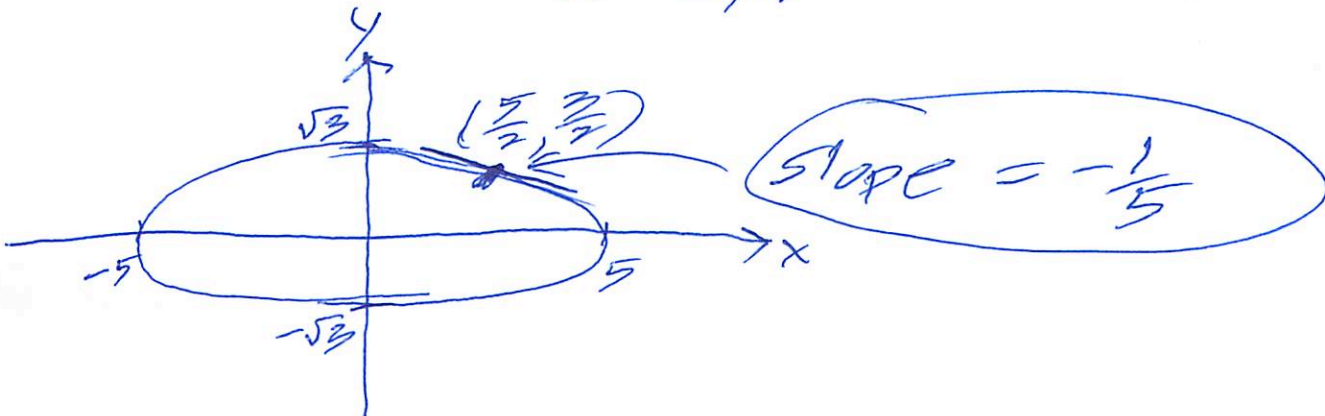
$$\frac{d}{dx} \left(\frac{x^2}{25} + \frac{y^2}{3} \right) = \frac{d}{dx} (1)$$

$$\frac{2}{25}x + \frac{2}{3}y \cdot \frac{dy}{dx} = 0$$

$$\frac{2}{3}y \frac{dy}{dx} = -\frac{2}{25}x$$

$$\frac{dy}{dx} = -\frac{3x}{25y}$$

$$\left. \frac{dy}{dx} \right|_{(x,y) = (\frac{5}{2}, \frac{3}{2})} = \frac{-3(\frac{5}{2})}{25(\frac{3}{2})} = -\frac{1}{5}$$



2. (2 points) Compute $g'(t)$ given that $g(t) = \sin^{-1}(5t)$.

$$g'(t) = \frac{1}{\sqrt{1-(5t)^2}} \cdot (5t)'$$
$$g'(t) = \frac{5}{\sqrt{1-25t^2}}$$

3. (4 points) Compute $\frac{dy}{dx}$ given that $\ln(x^2y^3) = x^3 + y^5$.

$$\ln(x^2) + \ln(y^3) = x^3 + y^5$$
$$2\ln x + 3\ln y = x^3 + y^5$$
$$\frac{d}{dx}(2\ln x + 3\ln y) = \frac{d}{dx}(x^3 + y^5)$$
$$2 \cdot \frac{1}{x} + 3 \cdot \frac{1}{y} \cdot \frac{dy}{dx} = 3x^2 + 5y^4 \cdot \frac{dy}{dx}$$
$$\frac{3}{y} \cdot \frac{dy}{dx} - 5y^4 \cdot \frac{dy}{dx} = 3x^2 - \frac{2}{x}$$
$$\frac{dy}{dx} \left(\frac{3}{y} - 5y^4 \right) = 3x^2 - \frac{2}{x}$$
$$\frac{dy}{dx} = \frac{3x^2 - \frac{2}{x}}{\frac{3}{y} - 5y^4}$$
$$= \frac{3x^3y - 2y}{3x - 5xy^5}$$