Name: Solutions

- No calculators allowed.
- Show sufficient work to justify each answer.
- You have 15 minutes for this quiz.

1. (3 points) Determine a formula for an exponential function given that its graph goes through the points \((-5,80), (0,20)\) and \((5,5)\).

\[
y = c \cdot a^x
\]

\((x, y) = (0, 20) \Rightarrow 20 = c \cdot a^0 \Rightarrow c = 20
\]

\[
y = 20 \cdot a^x
\]

\((x, y) = (5, 5) \Rightarrow 5 = 20 \cdot a^5
\]

\[
\Rightarrow \frac{1}{4} = a^5
\]

\[
\Rightarrow a = \left(\frac{1}{4}\right)^{\frac{1}{5}}
\]

\[
y = 20 \left(\frac{1}{4}\right)^{\frac{x}{5}}
\]

Note that \((x, y) = (-5, 80)\) satisfies this equation.
2. (3 points) Solve for $x$ in the equation below.

$x = e^{1+\ln(1-x)}$

$\begin{align*}
x &= e^1 \cdot e^{\ln(1-x)} \\
x &= e \cdot (1-x) \\
x &= e - ex \\
x + ex &= e \\
x(1+e) &= e \\
x &= \frac{e}{1+e} \quad \text{now check that this satisfies original equation}
\end{align*}$

3. (3 points) Given that $g(x) = \ln \left( 4 + \sqrt[3]{x} \right)$, find a formula for $g^{-1}(x)$.

$\begin{align*}
y &= \ln \left( 4 + \sqrt[3]{x} \right) \\
x &= \ln \left( 4 + \sqrt[3]{y} \right) \\
\text{(switch x & y)}
\end{align*}$

$\begin{align*}
e^x &= 4 + \sqrt[3]{y} \\
e^x - 4 &= \sqrt[3]{y} \\
\text{(solve for y)}
\end{align*}$

$g^{-1}(x) = (e^x - 4)^3$

4. (1 point) Suppose that $f$ is a one-to-one function which takes on the following values.

$f(-3) = 8, f(-2) = 3, f(-1) = 1/3, f(0) = -1/3, f(1) = -3, f(2) = -4, f(3) = -11$

What is the value of $f^{-1}(3)$?

Since $f(-2) = 3$, $f^{-1}(3) = -2$

switch roles of $x$ & $y$