

Name SOLUTIONS

- No calculators allowed.
- Show sufficient work to justify each answer.
- You have 15 minutes for this quiz.

1. (3 points) Determine a formula for an exponential function given that its graph goes through the points  $(-5, 80)$ ,  $(0, 20)$  and  $(5, 5)$ .

$$y = c \cdot a^x$$

$$(x, y) = (0, 20) \Rightarrow 20 = c \cdot a^0 \Rightarrow c = 20$$

$$y = 20 \cdot a^x$$

$$(x, y) = (5, 5) \Rightarrow 5 = 20 \cdot a^5$$

$$\Rightarrow \frac{1}{4} = a^5$$

$$\Rightarrow a = \left(\frac{1}{4}\right)^{1/5}$$

$$y = 20 \left(\frac{1}{4}\right)^{x/5}$$

$$y = 20 \left(\frac{1}{4}\right)^{x/5}$$

Note that  $(x, y) = (-5, 80)$  satisfies ~~the~~ this equation.

2. (3 points) Solve for  $x$  in the equation below.

$$x = e^{1+\ln(1-x)}$$

$$x = e^1 \cdot e^{\ln(1-x)}$$

$$x = e \cdot (1-x)$$

$$x = e - ex$$

$$x + ex = e$$

$$x(1+e) = e$$

$$x = \frac{e}{1+e}$$

→ now check that this satisfies original equation

3. (3 points) Given that  $g(x) = \ln(4 + \sqrt[3]{x})$ , find a formula for  $g^{-1}(x)$ .

$$y = \ln(4 + \sqrt[3]{x})$$

$$x = \ln(4 + \sqrt[3]{y})$$

(switch  $x$  &  $y$ )

$$e^x = 4 + \sqrt[3]{y}$$

$$e^x - 4 = \sqrt[3]{y}$$

$$y = (e^x - 4)^3$$

(solve for  $y$ )

$$g^{-1}(x) = (e^x - 4)^3$$

4. (1 point) Suppose that  $f$  is a one-to-one function which takes on the following values.

$$f(-3) = 8, f(-2) = 3, f(-1) = 1/3, f(0) = -1/3, f(1) = -3, f(2) = -4, f(3) = -11$$

What is the value of  $f^{-1}(3)$ ?

since  $f(-2) = 3$ ,  $f^{-1}(3) = -2$

switch roles of  $x$  &  $y$