Math 220 **Quiz 13 (take-home)** **Fall 2011**

Name SOLUTIONS

- You may work with other students in this class. However each student should write up solutions separately and independently – nobody should copy someone else's work.
- You may use your notes or the textbook.
- Computers are not allowed on any problem. You may use a calculator only for basic arithmetic.
- **You must show sufficient work to justify each answer.**
- The quiz should be turned in to your TA at the beginning of your discussion section meeting on Thursday, November 17th.
- Be sure that the pages are nicely stapled – do not just fold the corners.
- **Note to TAs and Tutors – you should not help students with these specific problems or go over solutions until after 4pm Thursday.**

1. (5 points) Determine an appropriate linear approximation of the function $f(x) = \sqrt{x}$ and use it to approximate $\sqrt{24.2}$. Write your answer in decimal form.

$f(x) = x^{1/2} \Rightarrow f(25) = 5$ point: $(25, 5)$

$f'(x) = \frac{1}{2}x^{-1/2} \Rightarrow f'(25) = \frac{1}{10}$ slope: $\frac{1}{10}$

tangent line

$y - 5 = \frac{1}{10}(x - 25) \Rightarrow y = 5 + \frac{1}{10}(x - 25)$

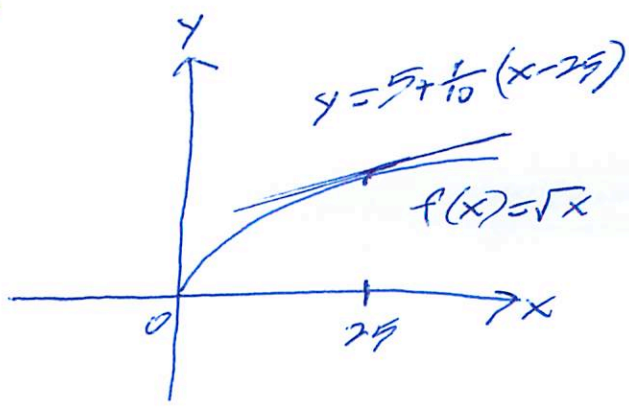
$\sqrt{x} \approx 5 + \frac{1}{10}(x - 25)$ for x near 25

$\sqrt{24.2} \approx 5 + \frac{1}{10}(24.2 - 25)$

$\sqrt{24.2} \approx 5 - 0.08$

$\sqrt{24.2} \approx 4.92$

overestimate



2. (5 points) The graphs of $f(x) = x^3$ and $g(x) = 3x+5$ have one intersection point. Determine the x-value for this intersection point using Newton's Method with an initial estimate of $x_1 = 2$. You should use this method 3 times in order to obtain estimates x_2, x_3 and x_4.

$x^3 = 3x+5$

$x^3 - 3x - 5 = 0$

Let $f(x) = x^3 - 3x - 5$.

Thus $f'(x) = 3x^2 - 3$.

Using Newton's Method to estimate a root of $f(x)$ we get:

$x_1 = 2$ (given)

$x_2 = x_1 - \frac{f(x_1)}{f'(x_1)} = 2 - \frac{f(2)}{f'(2)} = \frac{7}{3} = 2.\overline{3}$

$x_3 = x_2 - \frac{f(x_2)}{f'(x_2)} = 2.\overline{3} - \frac{f(2.\overline{3})}{f'(2.\overline{3})} = \frac{821}{360} = 2.280\overline{5}$

$x_4 = x_3 - \frac{f(x_3)}{f'(x_3)} = 2.280\overline{5} - \frac{f(2.280\overline{5})}{f'(2.280\overline{5})} \approx 2.27902$