Name SOLUTIONS

- You have 15 minutes
- No calculators
- Show sufficient work

1. (2 points) Find the average value of the function $f(x) = \frac{x}{\sqrt{x^2+9}}$ on the interval $[1, 4]$.

$$f_{ave} = \frac{1}{4-1}\int_1^4 \frac{x}{\sqrt{x^2+9}}\,dx$$

$$= \frac{1}{3}\int_{10}^{25} \frac{1/2}{\sqrt{u}}\,du$$

$$= \frac{1}{6}\int_{10}^{25} u^{-1/2}\,du$$

$$= \frac{1}{6}\left[2u^{1/2}\right]_{10}^{25}$$

$$= \frac{1}{6}\left[2\sqrt{25} - 2\sqrt{10}\right] = \frac{5-\sqrt{10}}{3}$$

substitution

$u = x^2+9$

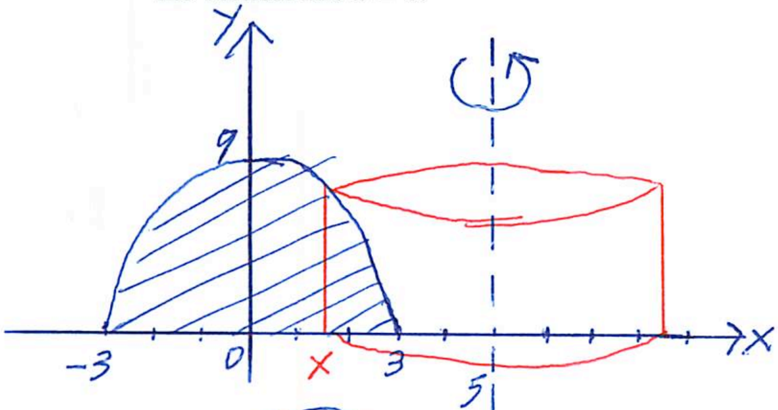
$du = 2x\,dx$

$\frac{1}{2}du = x\,dx$

$x=1 \Rightarrow u = 1^2+9 = 10$

$x=4 \Rightarrow u = 4^2+9 = 25$

2. (2 points) Let **R** be the finite region bounded by $y = 9 - x^2$ and the x-axis. Set up, <u>but do not evaluate</u>, a definite integral which represents the volume of the solid obtained when **R** is revolved around the vertical line $x = 5$.



$$V = \int_{-3}^{3} 2\pi(5-x)(9-x^2)\,dx$$

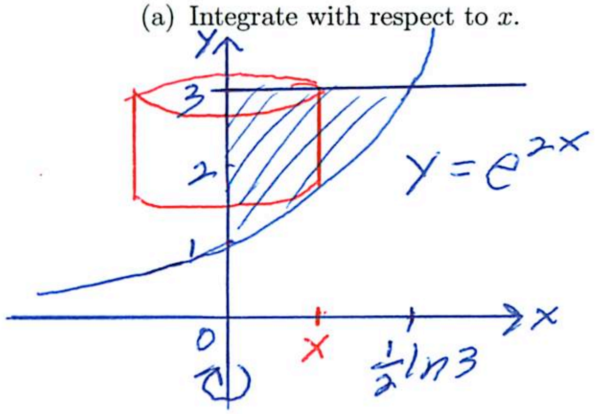
radius height

or

$$V = \int_0^9 \left(\pi\left(5-(-\sqrt{9-y})\right)^2 - \pi\left(5-\sqrt{9-y}\right)^2\right)dy$$

3. (3 points each) Let **R** be the finite region bounded by $y = e^{2x}$, $y = 3$ and the y-axis. In the following manner set up, but do not evaluate, definite integrals which represent the volume of the solid obtained when **R** is revolved around the y-axis.

(a) Integrate with respect to x.



intersection

$y=e^{2x}$ and $y=3$

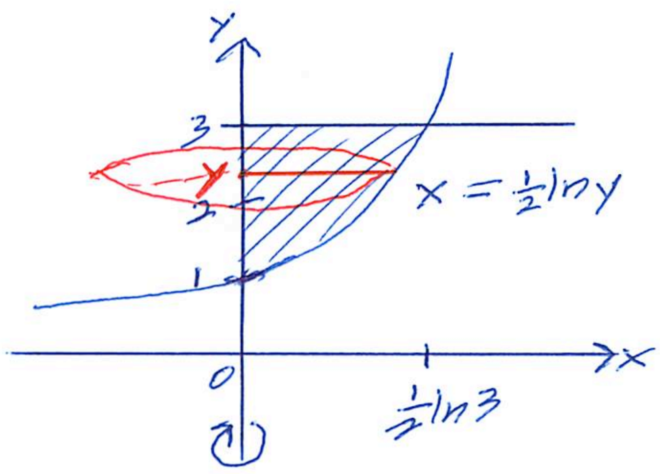
so $e^{2x}=3$

$2x=\ln 3$

$x=\frac{\ln 3}{2}$

$$V=\int_0^{\frac{1}{2}\ln 3} 2\pi \underset{\substack{\uparrow \\ \text{rad.}}}{x} \underset{\substack{\uparrow \\ \text{height}}}{(3-e^{2x})}\,dx$$

(b) Integrate with respect to y. (The integrands in parts (a) and (b) should be different.)



$x=\frac{1}{2}\ln y$ (we solved $y=e^{2x}$ for x)

$$V=\int_1^3 \pi\left(\tfrac{1}{2}\ln y\right)^2 dy$$