Name Solutions

- You have 15 minutes
- No calculators
- Show sufficient work

1. (2 points) Evaluate the integral $\int_1^2 \frac{24x}{1+4x^2}\,dx$.

$$\int_1^2 \frac{24x}{1+4x^2}\,dx = \int_5^{17} \frac{3}{u}\,du$$
$$= \left[3\ln|u|\right]_5^{17}$$
$$= 3\ln(17) - 3\ln(5)$$
$$= 3\ln\left(\frac{17}{5}\right)$$

substitution

$u = 1+4x^2$

$\frac{du}{dx} = 8x$

$du = 8x\,dx$

$3du = 24x\,dx$

$x=1 \Rightarrow u = 1+4(1)^2 = 5$

$x=2 \Rightarrow u = 1+4(2)^2 = 17$

2. (2 points) Evaluate the integral $\int \frac{12}{1+9x^2}\,dx$.

$$\int \frac{12}{1+9x^2}\,dx = \int \frac{12}{1+(3x)^2}\,dx$$
$$= \int \frac{4}{1+u^2}\,du$$
$$= 4\tan^{-1}(u) + C$$
$$= 4\tan^{-1}(3x) + C$$

substitution

$u = 3x$

$\frac{du}{dx} = 3$

$du = 3dx$

$4du = 12dx$

3. (2 points) Precisely state the *Mean Value Theorem.*

Let f be a function that satisfies the following hypotheses:

① f is continuous on the closed interval $[a,b]$.

② f is differentiable on the open interval (a,b).

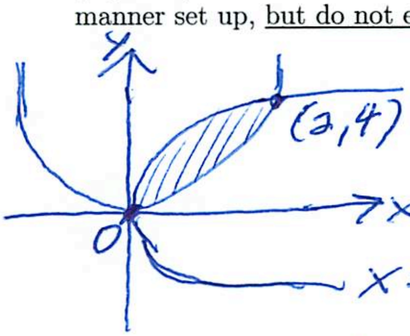
Then there is a number c in (a,b) such that

$$f'(c) = \frac{f(b)-f(a)}{b-a}$$

4. (4 points) Consider the finite region bounded by the curves $x = \frac{1}{8}y^2$ and $y = x^2$. In the following manner set up, but do not evaluate, definite integrals which represent the area of this region.

$y=x^2$

$x=\pm\sqrt{y}$



$x=\frac{1}{8}y^2$

$y=\pm\sqrt{8x}$

$x=\frac{1}{8}y^2 \Rightarrow y^2=8x$

$y=x^2 \Rightarrow y^2=x^4$

At intersection points,

$x^4=8x$

$x^4-8x=0$

$x(x^3-8)=0 \Rightarrow x=0$ or $x=2$

(a) Integrate with respect to x.

$$A=\int_0^2 (\sqrt{8x}-x^2)\,dx$$

(b) Integrate with respect to y. (The integrands in parts (a) and (b) should be different.)

$$A=\int_0^4 \left(\sqrt{y}-\frac{1}{8}y^2\right)dy$$