Math 220 (section ???) Quiz 10 Fall 2011

Name SOLUTIONS

- You have 15 minutes
- No calculators
- Show sufficient work

1. (3 points) Evaluate the following indefinite integral.

$$\int (10 + 3\tan^2 x)\, dx = \int (10+3(\sec^2 x-1))dx$$
$$= \int (7+3\sec^2 x)dx$$
$$= 7x + 3\tan x + C$$

2. (3 points) Sal loves blueberries. At 9:00 AM, she started eating some at a rate of $\frac{480}{t}$ blueberries per minute, where t denotes the number of minutes since 9:00 AM. What is the total number of blueberries that Sal ate between 10:00 AM and 11:00 AM ? Simplify your answer as much as possible without the use of a calculator.

Using "Net Change" Thm,

blueberries eaten between $t=60$ and $t=120$ $= \int_{60}^{120} \frac{480}{t}\, dt$

$$= \Big[480 \ln t\Big]_{60}^{120}$$
$$= 480\ln(120) - 480\ln(60)$$
$$= 480 \ln\left(\frac{120}{60}\right)$$
$$= 480\ln 2 \text{ blueberries}$$

Note: Although Sal really loves blueberries, this model is not ideal for small values of t since $\lim_{t\to 0^+} \frac{480}{t} = \infty$

using limits of

3. (2 points) Fill in the missing information to show that the given definite integral can be expressed as the limit of a Riemann sum. The only variables appearing in your limit should be n and k. You do not need to evaluate this limit.

right Riemann sums

$$\int_{-3}^{2}\frac{4}{7+x^2}\,dx = \lim_{n\to\infty}\sum_{k=1}^{n}\left[\frac{4}{7+\left(-3+k\cdot\frac{5}{n}\right)^2}\cdot\frac{5}{n}\right]$$

or

left Riemann sums

$$\int_{-3}^{2}\frac{4}{7+x^2}\,dx = \lim_{n\to\infty}\sum_{k=1}^{n}\left[\frac{4}{7+\left(-3+(k-1)\cdot\frac{5}{n}\right)^2}\cdot\frac{5}{n}\right]$$

or

midpoint Riemann sums

$$\int_{-3}^{2}\frac{4}{7+x^2}\,dx = \lim_{n\to\infty}\sum_{k=1}^{n}\left[\frac{4}{7+\left(-3+\left(k-\frac{1}{2}\right)\cdot\frac{5}{n}\right)^2}\cdot\frac{5}{n}\right]$$

4. (2 points) Suppose f is integrable on the interval $[1, 10]$. Given the following definite integrals, what is the value of $\int_3^6 f(x)\,dx$?

$$\int_1^6 f(x)\,dx = 5$$

$$\int_1^{10} f(x)\,dx = 17$$

$$\int_3^{10} f(x)\,dx = 8$$

$$\begin{aligned}
\int_3^6 f(x)\,dx &= \int_3^{10} f(x)\,dx + \int_{10}^{6} f(x)\,dx \\
&= \int_3^{10} f(x)\,dx + \left[\int_{10}^{1} f(x)\,dx + \int_1^6 f(x)\,dx\right] \\
&= \int_3^{10} f(x)\,dx + \left[-\int_1^{10} f(x)\,dx + \int_1^6 f(x)\,dx\right] \\
&= 8 + [-17 + 5] \\
&= -4
\end{aligned}$$

(there are other ways to obtain this answer)