MATH 220 Test 1 Fall 2011

Name SOLUTIONS UIN ____________

(circle your TA discussion section)

- ▷ **BD1**, TR 11:00-12:50, Vicki Reuter
- ▷ **BD2**, TR 9:00-9:50, Tom Mahoney
- ▷ **BD3**, TR 10:00-10:50, Kyle Knee
- ▷ **BD4**, TR 2:00-2:50, Neha Gupta
- ▷ **BD5**, TR 12:00-12:50, Nate Orlow
- ▷ **BD6**, TR 9:00-10:50, Ser-Wei Fu
- ▷ **BD7**, TR 3:00-3:50, Chayapa Darayon
- ▷ **BD8**, TR 1:00-1:50, Eliana Duarte

- ▷ **DD1**, TR 11:00-11:50, Nate Orlow
- ▷ **DD2**, TR 10:00-10:50, Santiago Camacho
- ▷ **DD3**, TR 9:00-9:50, Sarah Loeb
- ▷ **DD4**, TR 12:00-12:50, Lisa Hickok
- ▷ **DD5**, TR 1:00-1:50, Lisa Hickok
- ▷ **DD6**, TR 1:00-2:50, Jennifer Wise
- ▷ **DD7**, TR 8:00-8:50, Sarah Loeb
- ▷ **DD8**, TR 1:00-1:50, Abdulla Eid

- ▷ **AD1**, TR 11:00-11:50, Abdulla Eid
- ▷ **AD2**, TR 2:00-2:50, Ilkyoo Choi
- ▷ **AD3**, TR 1:00-1:50, Ilkyoo Choi
- ▷ **AD4**, TR 9:00-9:50, Michael Santana
- ▷ **AD5**, TR 3:00-3:50, Santiago Camacho
- ▷ **AD6**, TR 4:00-4:50, Joe Nance
- ▷ **AD7**, TR 3:00-3:50, Neha Gupta

- Sit in your assigned seat (shown below).
- Do not open this test booklet until I say *START*.
- Turn off all electronic devices and put away all items except a pen/pencil and an eraser.
- You must show sufficient work to justify each answer.
- While the test is in progress, we will not answer questions concerning the test material.
- Quit working and close this test booklet when I say *STOP*.
- Quickly turn in your test to me or a TA and show your Student ID.

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FRONT OF ROOM – 314 Altgeld Hall

1. (4 points each) Circle **true** if the given statement is always true. Otherwise circle **false**.

(a) Given a function g, if $|g(x)| \le x^4$ for all x then $\lim_{x\to 0} g(x) = 0$.

true or **false** ?

$-x^4 \le g(x) \le x^4$

$\lim_{x\to 0}(-x^4) = 0$ and $\lim_{x\to 0}(x^4) = 0$

by squeeze theorem, $\lim_{x\to 0} g(x) = 0$

(b) If the point $\left(-4, \frac{1}{4}\right)$ is on the graph of an odd function g then $\left(-\frac{1}{4}, 4\right)$ is another point on the graph of g.

true or **false** ?

If $(-4, \frac{1}{4})$ is on graph, then $(4, -\frac{1}{4})$ is also on graph, but not necessarily $(-\frac{1}{4}, 4)$. Think about $g(x) = -\frac{1}{16}x$

(c) Given a function g, if $\underbrace{\lim_{x\to 4} \frac{g(x) - g(4)}{x - 4} \text{ exists}}$ then g is continuous at 4.

true or **false** ?

This shows g is differentiable at 4, but differentiability implies continuity.

(d) If a function g is continuous at 0 then $\lim_{x\to 0} g(x) = 0$.

true or **false** ?

If g is continuous at 0, then $\lim_{x\to 0} g(x) = g(0)$, but $g(0)$ is not always equal to 0. Look at $g(x) = x^2+1$.

(e) A function which is continuous at a point a must also be differentiable at a.

true or **false** ?

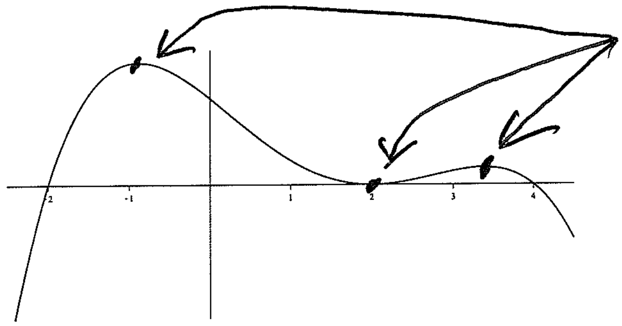
$f(x) = |x|$ is continuous but not differentiable at 0.

(f) If a function g is one-to-one then $g(1) = 1$.

true or **false** ?

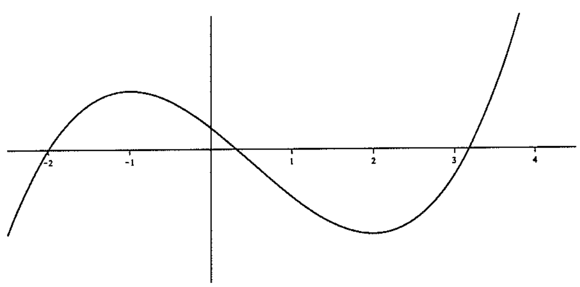
$g(x) = 2x$ is one-to-one but $g(1) = 2$.

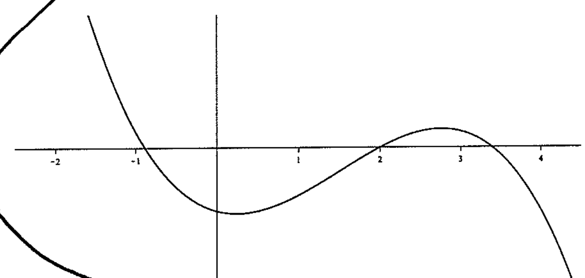
2. (6 points) Here is the graph of $y = f(x)$.

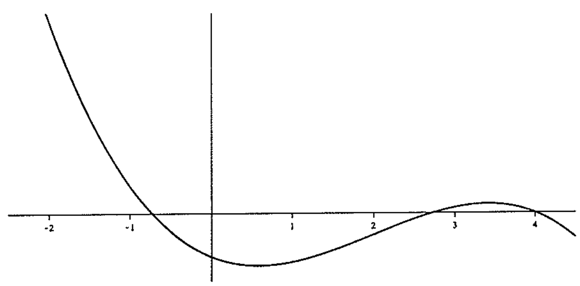


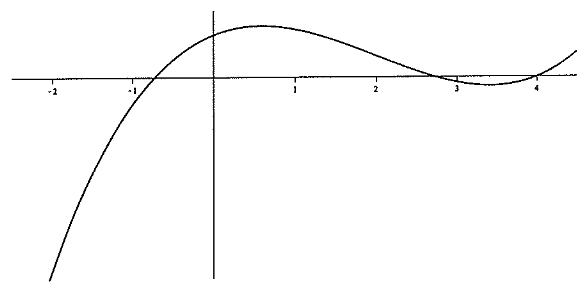
$f' = 0$

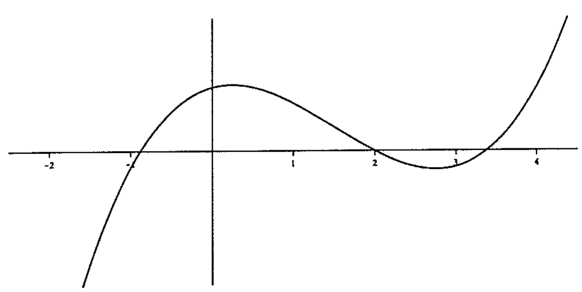
Circle the graph of $y = f'(x)$, given that it is one of the 6 choices below.

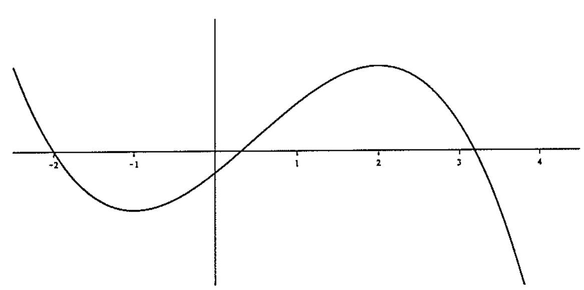












3. (12 points) Let $f(x) = 4x^3 + 2$. Use the definition of a derivative as a limit to prove that $f'(x) = 12x^2$

Show each step in your calculation and be sure to use proper terminology in each step of your proof.

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$$= \lim_{h \to 0} \frac{4(x+h)^3 + 2 - (4x^3 + 2)}{h}$$

$$= \lim_{h \to 0} \frac{4(x^3 + 3x^2h + 3xh^2 + h^3) + 2 - 4x^3 - 2}{h}$$

$$= \lim_{h \to 0} \frac{4x^3 + 12x^2h + 12xh^2 + 4h^3 + 2 - 4x^3 - 2}{h}$$

$$= \lim_{h \to 0} \frac{h(12x^2 + 12xh + 4h^2)}{h}$$

$$= \lim_{h \to 0} (12x^2 + 12xh + 4h^2)$$

$$= 12x^2$$

4. (6 points) Suppose that f and g are one-to-one functions which take on the following values.

$$f(-2) = 2, \quad f(-1) = 1/2, \quad f(0) = -1/2, \quad f(1) = -2, \quad f(2) = -4$$
$$g(-2) = -4, \quad g(-1) = -2, \quad g(0) = -1/2, \quad g(1) = 1/2, \quad g(2) = 2$$

What is the value of $f^{-1}\left(g^{-1}(-4)\right)$?

$f^{-1}(g^{-1}(-4)) = f^{-1}(-2)$ since $g(-2) = -4$

$= 1$ since $f(1) = -2$

5. (4 points each) State the domain of each function.

(a) $f(x) = \cos^{-1} x$

domain $[-1, 1]$

(b) $g(x) = \dfrac{8-x}{\ln(x-4)}$

$x - 4 > 0 \Rightarrow x > 4$

$\ln(x-4) \neq 0 \Rightarrow x - 4 \neq 1 \Rightarrow x \neq 5$

domain: $(4, 5) \cup (5, \infty)$

(c) $h(x) = \sqrt{x^2 + 9}$

domain: $(-\infty, \infty)$

6. (10 points) Solve for x in the equation below.

$$\ln(x-4)+\ln(x-1)=2\ln(5-x)$$

$$\ln((x-4)(x-1)) = \ln((5-x)^2)$$

$$(x-4)(x-1) = (5-x)^2$$

$$x^2-5x+4 = 25-10x+x^2$$

$$-5x+4 = 25-10x$$

$$5x = 21$$

$$x = \frac{21}{5}$$

check that $x=\frac{21}{5}$ is a solution ✓

7. (6 points each) Evaluate the following limits. Show sufficient justification for each answer. An answer of 'does not exist' is not sufficient. For infinite limits you must state if it is ∞ or $-\infty$.

(a) $\displaystyle\lim_{x\to\infty}\frac{(3x+1)^2}{2x^2+5}$ $\displaystyle = \lim_{x\to\infty}\frac{9x^2+6x+1}{2x^2+5}$

$$= \lim_{x\to\infty}\frac{x^2\left(9+\frac{6}{x}+\frac{1}{x^2}\right)}{x^2\left(2+\frac{5}{x^2}\right)}$$

$$= \lim_{x\to\infty}\frac{9+\frac{6}{x}+\frac{1}{x^2}}{2+\frac{5}{x^2}}$$

$$= \frac{9}{2}$$

(b) $\displaystyle\lim_{x\to 0}\frac{5x^2+2x+3}{7x^2+4}$ $\displaystyle = \frac{5(0)^2+2(0)+3}{7(0)^2+4}$

$$= \frac{3}{4}$$

(c) $\lim_{x\to 1} \frac{\sqrt{9x}-3}{x-1} = \lim_{x\to 1} \frac{(\sqrt{9x}-3)(\sqrt{9x}+3)}{(x-1)(\sqrt{9x}+3)}$

$$= \lim_{x\to 1} \frac{9x-9}{(x-1)(\sqrt{9x}+3)}$$

$$= \lim_{x\to 1} \frac{9(x-1)}{(x-1)(\sqrt{9x}+3)}$$

$$= \lim_{x\to 1} \frac{9}{\sqrt{9x}+3} = \frac{9}{\sqrt{9}+3} = \frac{9}{6} = \frac{3}{2}$$

(d) $\lim_{x\to 0} \left(\frac{1-(\cos x + \sin x)^2}{10x \cos x} \right) = \lim_{x\to 0} \left(\frac{1-(\cos^2 x + 2\sin x\cos x + \sin^2 x)}{10x\cos x} \right)$

$$= \lim_{x\to 0} \left(\frac{1-(2\sin x\cos x + \cos^2 x + \sin^2 x)}{10x\cos x} \right)$$

$$= \lim_{x\to 0} \left(\frac{1-(2\sin x\cos x + 1)}{10x\cos x} \right)$$

$$= \lim_{x\to 0} \frac{-2\sin x\cos x}{10x\cos x}$$

$$= \lim_{x\to 0} \left(\frac{-1}{5} \cdot \frac{\sin x}{x} \right) = \frac{-1}{5} \cdot 1 = \frac{-1}{5}$$

(e) $\lim_{x\to 4^-} \frac{\ln(x/8)}{\ln(x/4)}$ $\;\; \ln(x/8) \to \ln(1/2)$, $\;\; \ln(x/4) \to 0^-$

$\ln(x/8) \to \ln(1/2)$, a neg. value, as $x \to 4^-$

$\ln(x/4) \to 0^-$ as $x \to 4^-$

Thus $\lim_{x\to 4^-} \frac{\ln(x/8)}{\ln(x/4)} = \infty$

Students – do not write on this page!

1. (24 points) ____________________

2. (6 points) ____________________

3. (12 points) ____________________

4. (6 points) ____________________

5. (12 points) ____________________

6. (10 points) ____________________

7a. (6 points) ____________________

7b. (6 points) ____________________

7c. (6 points) ____________________

7d. (6 points) ____________________

7e. (6 points) ____________________

TOTAL (100 points) ____________