Math 220 (section ???) Quiz 9 Fall 2010

Name SOLUTIONS

You have 15 minutes for this quiz – no calculators allowed.

1. (3 points) Find the most general antiderivative of the function $f(x) = 10x^2(2x-3)^2$.

$f(x) = 10x^2(2x-3)^2$

$f(x) = 10x^2(4x^2-12x+9)$

$f(x) = 40x^4 - 120x^3 + 90x^2$

$F(x) = 40\left(\frac{1}{5}x^5\right) - 120\left(\frac{1}{4}x^4\right) + 90\left(\frac{1}{3}x^3\right) + C$

$F(x) = 8x^5 - 30x^4 + 30x^3 + C$

where C is an arbitrary constant

2. (4 points) Find a formula for $f(x)$ given that $f''(x) = 4\cos x$, $f(0) = 10$ and $f'(0) = 5$.

$f''(x) = 4\cos x$

$f'(x) = 4\sin x + C_1$

letting $x = 0$ we get

$5 = 4\sin(0) + C_1 \Rightarrow C_1 = 5$

$f'(x) = 4\sin x + 5$

$f(x) = -4\cos x + 5x + C_2$

letting $x = 0$ we get

$10 = -4\cos(0) + 5(0) + C_2 \Rightarrow C_2 = 14$

$f(x) = -4\cos x + 5x + 14$

3. (3 points) The area between the x-axis and the graph of $f(x) = x^2 \ln x$ on the interval $[5, 15]$ can be written as a limit. Fill in the missing information in this limit so that the only variables appearing are n and k. You do not need to evaluate this limit.

$$AREA = \lim_{n\to\infty} \sum_{k=1}^{n} \left[\left(5+\frac{10k}{n}\right)^2 \ln\left(5+\frac{10k}{n}\right)\left(\frac{10}{n}\right) \right]$$

$$A = \lim_{n\to\infty} \sum_{k=1}^{n} f(x_k)\Delta x$$

where $\Delta x = \frac{b-a}{n} = \frac{15-5}{n} = \frac{10}{n}$

and $x_k = a + k\Delta x = 5 + k\left(\frac{10}{n}\right)$

so

$$A = \lim_{n\to\infty} \sum_{k=1}^{n} \left(5+\frac{10k}{n}\right)^2 \ln\left(5+\frac{10k}{n}\right)\left(\frac{10}{n}\right)$$