Math 220 | Quiz 8 (take-home) | Fall 2010

Your Name SOLUTIONS

TA's Name ______________________

Discussion Section ______________________
(list either section number or meeting times)

- You may work with other students in this class. However each student should write up solutions separately and independently – nobody should copy someone else's work.
- You may use your notes or the textbook.
- No calculators or computers are allowed on any problem.
- You must show sufficient work to justify each answer.
- The quiz should be turned in to your TA by 4pm Friday, October 15th. You are allowed to turn it in early when you next see your TA. Although most TA mailboxes are in 250 Altgeld Hall, your TA may inform you about a preferred way to submit your quiz.
- Be sure that the pages are nicely stapled – do not just fold the corners.
- Note to TAs – you should not help students with these specific problems or go over solutions until after 4pm Friday.

1. (3 points) Find all inflection points on the graph of $f(x) = 3x^5 - 10x^4 + 10x^3 - 40x + 50$.

$f'(x) = 15x^4 - 40x^3 + 30x^2 - 40$

$f''(x) = 60x^3 - 120x^2 + 60x$

$= 60x(x^2 - 2x + 1)$

$= 60x(x-1)^2$

values of $f''(x)$

$- - - 0 + + + 0 + + +$

$\longrightarrow x$

$0 \qquad 1$

$\to$ f changes concavity at $x=0$, so there is an inflection point at $(0, f(0)) = (0, 50)$

2. (3 points) Determine the x-value at which the function $f(x) = \dfrac{e^x}{e^{2x}+5}$ attains its absolute maximum value.

$$f'(x) = \frac{(e^x)'(e^{2x}+5) - (e^x)(e^{2x}+5)'}{(e^{2x}+5)^2}$$

$$f'(x) = \frac{e^x(e^{2x}+5) - e^x(2e^{2x})}{(e^{2x}+5)^2}$$

$$f'(x) = \frac{e^x(e^{2x}+5-2e^{2x})}{(e^{2x}+5)^2}$$

$$f'(x) = \frac{e^x(5-e^{2x})}{(e^{2x}+5)^2}$$

e^x and $(e^{2x}+5)^2$ are always positive. $f'(x) = 0$ when $5 - e^{2x} = 0$. Solving for x gives $x = \frac{\ln 5}{2}$

values of $f'(x)$

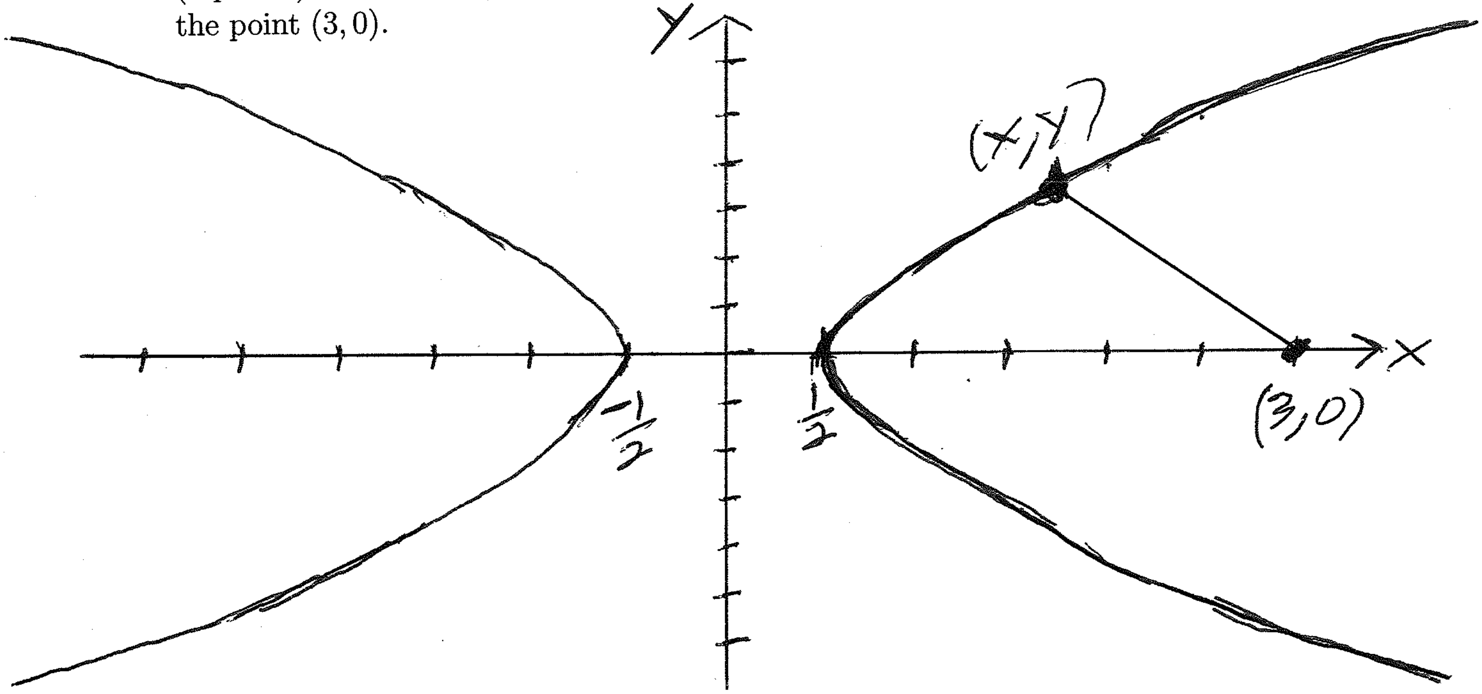
$+ + + 0 - - - -$

$\longrightarrow x$

$(\ln 5)/2$

max value occurs when $x = \frac{\ln 5}{2}$

3. (4 points) Find the points on the graph of the hyperbola $4x^2 - y^2 = 1$ which are closest to the point $(3, 0)$.



Let (x,y) be an arbitrary point on the hyperbola. It is clear that the point closest to $(3,0)$ is on the right branch of this hyperbola. Thus we need to minimize the distance between (x,y) and $(3,0)$ for x in the interval $[\frac{1}{2}, \infty)$

$D = \sqrt{(x-3)^2 + (y-0)^2}$

$D = \sqrt{(x-3)^2 + y^2}$

$D = \sqrt{(x-3)^2 + 4x^2 - 1}$ since $4x^2 - y^2 = 1$ or $y^2 = 4x^2 - 1$ for each point (x,y) on the hyperbola.

$$D' = \frac{1}{2}\left((x-3)^2+4x^2-1\right)^{-1/2} \cdot \left(2(x-3)+8x\right)$$

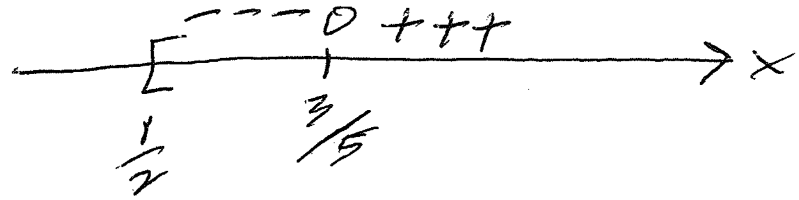
$$D' = \frac{10x-6}{2\sqrt{(x-3)^2+4x^2-1}}$$

$$D' = \frac{5x-3}{\sqrt{(x-3)^2+4x^2-1}}$$

(do you see that the denominator is positive for all x?)

$D' = 0$ when $5x-3=0$. Solving for x gives $x = 3/5$

values of D'



minimum distance occurs when $x = 3/5$. From $4x^2-y^2=1$ we get $4\left(\frac{3}{5}\right)^2-y^2=1$. Solving for y gives $y = \pm\frac{\sqrt{11}}{5}$. The points closest to $(3,0)$ are $\left(\frac{3}{5}, -\frac{\sqrt{11}}{5}\right)$ and $\left(\frac{3}{5}, \frac{\sqrt{11}}{5}\right)$

The distance is $D = \frac{\sqrt{155}}{5} \approx 2.49$

NOTE

TO SIMPLIFY THE ALGEBRA, ONE COULD CHOOSE TO MINIMIZE THE SQUARE OF THE DISTANCE.

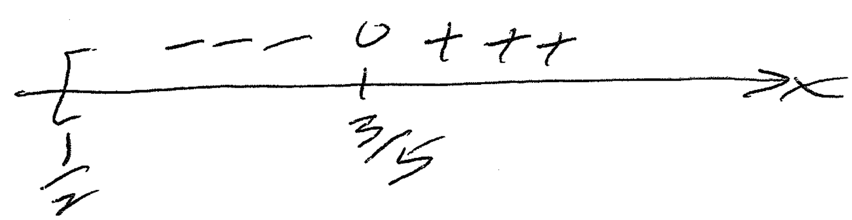
$$f(x) = D^2 = (x-3)^2 + 4x^2 - 1$$

so $f'(x) = 2(x-3) + 8x$

$$f'(x) = 10x - 6$$

$f'(x) = 0$ when $x = 3/5$

values of $f'(x)$



minimum distance when $x = \frac{3}{5}$

$\vdots$