Math 220 Quiz 7 (take-home) Fall 2010

Your Name SOLUTIONS

TA's Name _____

Discussion Section _____
(list either section number or meeting times)

- You may work with other students in this class. However each student should write up solutions separately and independently – nobody should copy someone else's work.
- You may use your notes or the textbook.
- No calculators or computers are allowed on any problem.
- You must show sufficient work to justify each answer.
- The quiz should be turned in to your TA at the beginning of your discussion section meeting on Monday, October 11 (Merit sections) or Tuesday, October 12 (other sections).
- Be sure that the pages are nicely stapled – do not just fold the corners.
- Note to TA's – you should not help students with these specific problems or go over solutions until after 4pm Tuesday.

1. (3 points) Find the equation of the line tangent to the curve given by $x^2 + 6xy^2 + y^3 = 17$ at the point $(2, 1)$.

$$\frac{d}{dx}(x^2+6xy^2+y^3) = \frac{d}{dx}(17)$$

$$2x + \frac{d}{dx}(6x)y^2 + 6x\frac{d}{dx}(y^2) + 3y^2\frac{dy}{dx} = 0$$

$$2x + 6y^2 + 6x\left(2y\frac{dy}{dx}\right) + 3y^2\frac{dy}{dx} = 0$$

$$\frac{dy}{dx}(12xy + 3y^2) = -2x - 6y^2$$

$$\frac{dy}{dx} = \frac{-2x-6y^2}{12xy+3y^2}$$

$$\left.\frac{dy}{dx}\right|_{(x,y)=(2,1)} = \frac{-2(2)-6(1)^2}{12(2)(1)+3(1)^2} = \frac{-10}{27}$$

So $y - 1 = \frac{-10}{27}(x-2)$ or $y = \frac{-10}{27}x + \frac{47}{27}$

2. (2 points) For the given function, use logarithmic differentiation to find a formula for $\frac{dy}{dx}$ written in terms of x.

$$y = \left(x^5+1\right)^{3x^2}$$

$$\ln y = \ln\left((x^5+1)^{3x^2}\right)$$

$$\ln y = 3x^2 \ln(x^5+1)$$

$$\frac{d}{dx}(\ln y) = \frac{d}{dx}(3x^2\ln(x^5+1))$$

$$\frac{1}{y}\cdot\frac{dy}{dx} = 6x\ln(x^5+1) + 3x^2\cdot\frac{1}{x^5+1}\cdot 5x^4$$

$$\frac{dy}{dx} = y\left(6x\ln(x^5+1) + \frac{15x^6}{x^5+1}\right)$$

$$\frac{dy}{dx} = (x^5+1)^{3x^2}\left(6x\ln(x^5+1) + \frac{15x^6}{x^5+1}\right)$$

3. (2 points) Given that $\frac{dw}{dr} = 0.25w$ and $w(4) = 3$, find a formula for w as a function of r.

$w = Ce^{0.25r}$

$3 = Ce^{0.25(4)}$

$3 = Ce$

$C = \frac{3}{e}$

$w = \left(\frac{3}{e}\right)e^{0.25r}$

$\boxed{w = 3e^{0.25r-1}}$

4. (3 points) A ball is thrown straight up from an initial height of 8 feet above the ground. Until the ball hits the ground, the function $h = -16\left(t - \frac{1}{2}\right)^2 + 12$ represents the ball's height in feet above ground level t seconds after it was thrown. What is the velocity of the ball when it hits the ground?

when it hits the ground, $h = 0$ so

$0 = -16\left(t - \frac{1}{2}\right)^2 + 12$

$16\left(t - \frac{1}{2}\right)^2 = 12$

$\left(t - \frac{1}{2}\right)^2 = \frac{12}{16} = \frac{3}{4}$

$t - \frac{1}{2} = \pm\sqrt{\frac{3}{4}} = \pm\frac{\sqrt{3}}{2}$

$t = \frac{1}{2} \pm \frac{\sqrt{3}}{2}$

but $t > 0$ so

$\boxed{t = \frac{1}{2} + \frac{\sqrt{3}}{2}}$

$h'(t) = -32\left(t - \frac{1}{2}\right)$

$h'\left(\frac{1}{2} + \frac{\sqrt{3}}{2}\right) = -32\left(\frac{\sqrt{3}}{2}\right) = \boxed{-16\sqrt{3} \text{ ft/sec}}$