1. (2 points each) Differentiate the following functions.

(a) \( W = 5 \cot x + 4 \csc x \)

\[
\frac{dW}{dx} = 5 \left( -\csc^2 x \right) + 4 \left( -\csc x \cot x \right)
\]

\[
\frac{dW}{dx} = -5 \csc^2 x - 4 \csc x \cot x
\]

An alternate approach is to rewrite everything in terms of sine and cosine to get

\[
W = 5 \left( \frac{\cos x}{\sin x} \right) + 4 \left( \frac{1}{\sin x} \right)
\]

\[
W = \frac{5 \cos x + 4}{\sin x}
\]

From the quotient rule we get

\[
\frac{dW}{dx} = \frac{(5 \cos x + 4)'(\sin x) - (5 \cos x + 4)(\sin x)'}{(\sin x)^2}
\]

\[
\frac{dW}{dx} = \frac{(-5 \sin x)(\sin x) - (5 \cos x + 4)(\cos x)}{\sin^2 x}
\]

\[
\frac{dW}{dx} = \frac{-5 \sin^2 x - 5 \cos^2 x - 4 \cos x}{\sin^2 x}
\]

\[
\frac{dW}{dx} = \frac{-5(\sin^2 x + \cos^2 x) - 4 \cos x}{\sin^2 x}
\]

\[
\frac{dW}{dx} = \frac{-5 - 4 \cos x}{\sin^2 x}
\]

(b) \( g(x) = x^3 e^x \sin x \)

\( g(x) = x^3 e^x \cdot \sin x \)

\( g'(x) = \left( x^3 e^x \right)'(\sin x) + \left( x^3 e^x \right)(\sin x)' \)

\( g'(x) = \left( \left( x^3 \right)'(e^x) + \left( x^3 \right)(e^x)' \right)(\sin x) + \left( x^3 e^x \right)(\sin x)' \)

\( g'(x) = \left( x^3 \right)'(e^x)(\sin x) + \left( x^3 \right)(e^x)'(\sin x) + \left( x^3 \right)(e^x)(\sin x)' \)

The above shows how the product rule applies to a product of three functions. Can you see how it would generalize to a product of an arbitrary number of functions?

The final answer here is

\( g'(x) = 3x^2 e^x \sin x + x^3 e^x \sin x + x^3 e^x \cos x \)
2. (3 points) What is the equation of the line tangent to the graph of the following function at its $y$-intercept?

$$y = 3 \tan x + 5x + 2$$

The $y$-intercept has coordinates $(0, y(0)) = (0, 2)$.

Since $y' = 3 \sec^2 x + 5$, the slope at $(0, 2)$ is $y'(0) = 3 \sec^2 (0) + 5 = 3/\cos^2 (0) + 5 = 3 + 5 = 8$.

The equation of the line which goes through the point $(0, 2)$ and has slope 8 is $y = 8x + 2$.

3. (3 points) Find the second derivative $f''(t)$ of the following function.

$$f(t) = \sec t$$

$$f'(t) = \sec t \tan t$$

$$f''(t) = (\sec t)'(\tan t) + (\sec t)(\tan t)'$$

$$f''(t) = (\sec t \tan t) (\tan t) + (\sec t) \left( \sec^2 t \right)$$

$$f''(t) = \sec t \tan^2 t + \sec^3 t$$

An alternate approach is to rewrite everything in terms of sine and cosine and use the quotient rule twice.