Math 220 (section ???) **Quiz 5 Solutions** **Fall 2010**

1. (2 points each) Differentiate the following functions.

   (a) $W = 5\cot x + 4\csc x$

   $$\frac{dW}{dx} = 5\left(-\csc^2 x\right) + 4\left(-\csc x \cot x\right)$$
   $$\frac{dW}{dx} = -5\csc^2 x - 4\csc x\cot x$$

   An alternate approach is to rewrite everything in terms of sine and cosine to get
   $W = 5\left(\frac{\cos x}{\sin x}\right) + 4\left(\frac{1}{\sin x}\right)$
   $$W = \frac{5\cos x + 4}{\sin x}$$

   From the quotient rule we get
   $$\frac{dW}{dx} = \frac{(5\cos x + 4)'(\sin x) - (5\cos x + 4)(\sin x)'}{(\sin x)^2}$$
   $$\frac{dW}{dx} = \frac{(-5\sin x)(\sin x) - (5\cos x + 4)(\cos x)}{\sin^2 x}$$
   $$\frac{dW}{dx} = \frac{-5\sin^2 x - 5\cos^2 x - 4\cos x}{\sin^2 x}$$
   $$\frac{dW}{dx} = \frac{-5(\sin^2 x + \cos^2 x) - 4\cos x}{\sin^2 x}$$
   $$\frac{dW}{dx} = \frac{-5 - 4\cos x}{\sin^2 x}$$

   (b) $g(x) = x^3 e^x \sin x$

   $g(x) = x^3 e^x \cdot \sin x$

   $g'(x) = \left(x^3 e^x\right)'(\sin x) + \left(x^3 e^x\right)(\sin x)'$

   $g'(x) = \left(\left(x^3\right)'(e^x) + \left(x^3\right)(e^x)'\right)(\sin x) + \left(x^3 e^x\right)(\sin x)'$

   $g'(x) = \left(x^3\right)'(e^x)(\sin x) + \left(x^3\right)(e^x)'(\sin x) + \left(x^3\right)(e^x)(\sin x)'$

   The above shows how the product rule applies to a product of three functions. Can you see how it would generalize to a product of an arbitrary number of functions?

   The final answer here is

   $g'(x) = 3x^2 e^x \sin x + x^3 e^x \sin x + x^3 e^x \cos x$

2. (3 points) What is the equation of the line tangent to the graph of the following function at its y-intercept?

$$y = 3\tan x + 5x + 2$$

The y-intercept has coordinates $(0, y(0)) = (0, 2)$.

Since $y' = 3\sec^2 x + 5$, the slope at $(0, 2)$ is $y'(0) = 3\sec^2(0) + 5 = 3/\cos^2(0) + 5 = 3 + 5 = 8$.

The equation of the line which goes through the point $(0, 2)$ and has slope 8 is $y = 8x + 2$.

3. (3 points) Find the second derivative $f''(t)$ of the following function.

$f(t) = \sec t$

$f'(t) = \sec t \tan t$

$f''(t) = (\sec t)' (\tan t) + (\sec t)(\tan t)'$

$f''(t) = (\sec t \tan t)(\tan t) + (\sec t)\left(\sec^2 t\right)$

$f''(t) = \sec t \tan^2 t + \sec^3 t$

An alternate approach is to rewrite everything in terms of sine and cosine and use the quotient rule twice.