SOLUTIONS

- No calculators allowed.
- Show sufficient work to justify each answer.
- You have 20 minutes for this quiz.

1. (2 points) Determine the values of c and d so that \( f(x) \) is continuous throughout its domain.

\[
f(x) = \begin{cases} 
3x - 5 & \text{for } x < -2 \\
cx + d & \text{for } -2 \leq x \leq 3 \\
5 - 2x & \text{for } x > 3 
\end{cases}
\]

For \( x < -2 \), \(-2 < x < 3 \), and \( x > 3 \), \( f \) is continuous since it is a polynomial on each of these intervals. Now we need to choose \( c \) and \( d \) so that \( f \) is also continuous at \( x = -2 \) and \( x = 3 \). Thus we need \( \lim_{x \to -2} f(x) = f(-2) \) and \( \lim_{x \to 3} f(x) = f(3) \).

\[
\lim_{x \to -2} f(x) = \lim_{x \to -2} (3x - 5) = -11
\]

\[
\lim_{x \to -2} f(x) = \lim_{x \to -2} (cx + d) = -2c + d
\]

\[
f(-2) = -2c + d
\]

\[
\lim_{x \to 3} f(x) = \lim_{x \to 3} (cx + d) = 3c + d
\]

\[
\lim_{x \to 3} f(x) = \lim_{x \to 3} (5 - 2x) = -1
\]

\[
f(3) = 3c + d
\]

Solving the two equations above gives \( c = 2 \) and \( d = -7 \).
2. (2 points each) Evaluate the following limits.

(a) \[ \lim_{x \to 0^+} \frac{5 - x^2}{1 - e^x} = -\infty \]

Note that \( e^x > 1 \) for all \( x > 0 \), so \( 1 - e^x < 0 \) for all \( x > 0 \). Thus \( 1 - e^x \to 0^- \) as \( x \to 0^+ \).

(b) \[ \lim_{x \to 5} \frac{x^2 - 25}{x^2 - 3x - 10} = \lim_{x \to 5} \frac{(x-5)(x+5)}{(x-5)(x+2)} \]

\[ = \lim_{x \to 5} \frac{x+5}{x+2} \]

\[ = \frac{10}{7} \]

(c) \[ \lim_{x \to \infty} \frac{6x^2 + 5}{x^2 + 3} = \lim_{x \to \infty} \frac{(6x^2 + 5)(\frac{1}{x^2})}{(x^2 + 3)(\frac{1}{x^2})} \]

\[ = \lim_{x \to \infty} \frac{6 + 5/x^2}{1 + 3/x^2} \]

\[ = \frac{6}{3} \]

\[ = 2 \]
3. (2 points) A function $f$ satisfies the following inequality for all $x \neq 0$.

$$\frac{3x + 2\sin x}{2x} \leq f(x) \leq \frac{7x - 2\sin x}{2x}$$

Determine $\lim_{x \to 0} f(x)$.

$$\lim_{x \to 0} \frac{3x + 2\sin x}{2x} = \lim_{x \to 0} \left( \frac{3x}{2x} + \frac{2\sin x}{2x} \right)$$

$$= \lim_{x \to 0} \frac{3}{2} + \lim_{x \to 0} \frac{2\sin x}{x}$$

$$= \frac{3}{2} + 1 = \frac{5}{2}$$

$$\lim_{x \to 0} \frac{7x - 2\sin x}{2x} = \lim_{x \to 0} \left( \frac{7x}{2x} - \frac{2\sin x}{2x} \right)$$

$$= \lim_{x \to 0} \left( \frac{7}{2} - \frac{2\sin x}{x} \right)$$

$$= \lim_{x \to 0} \left( \frac{7}{2} \right) - \lim_{x \to 0} \frac{\sin x}{x}$$

$$= \frac{7}{2} - 1 = \frac{5}{2}$$

By the Squeeze Theorem,

$$\lim_{x \to 0} f(x) = \frac{5}{2}$$