Math 220 (section ???) Quiz 3 Fall 2010

Name SOLUTIONS

- No calculators allowed.
- Show sufficient work to justify each answer.
- You have 20 minutes for this quiz.

1. (2 points) Determine the values of c and d so that $f(x)$ is continuous throughout its domain.

$$f(x) = \begin{cases} 3x-5 & \text{for} \quad x < -2 \\ cx+d & \text{for} \quad -2 \le x \le 3 \\ 5-2x & \text{for} \quad x > 3 \end{cases}$$

For $x<-2$, $-2<x<3$, and $x>3$, f is continuous since it is a polynomial on each of these intervals. Now we need to choose c and d so that f is also continuous at $x=-2$ and $x=3$. Thus we need $\lim\limits_{x\to -2} f(x) = f(-2)$ and $\lim\limits_{x\to 3} f(x) = f(3)$.

$\lim\limits_{x\to -2^-} f(x) = \lim\limits_{x\to -2^-} (3x-5) = -11$

$\lim\limits_{x\to -2^+} f(x) = \lim\limits_{x\to -2^+} (cx+d) = -2c+d$

$f(-2) = -2c+d$

$-11 = -2c+d$

$\lim\limits_{x\to 3^-} f(x) = \lim\limits_{x\to 3^-} (cx+d) = 3c+d$

$\lim\limits_{x\to 3^+} f(x) = \lim\limits_{x\to 3^+} (5-2x) = -1$

$f(3) = 3c+d$

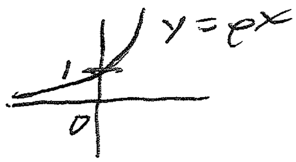
$-1 = 3c+d$

solving the two equations above gives $c=2$ and $d=-7$

2. (2 points each) Evaluate the following limits.

(a) $\lim\limits_{x\to 0^+} \frac{5-x^2}{1-e^x}$ $= -\infty$

numerator $\to 5$, denominator $\to 0^-$



note that $e^x > 1$ for all $x > 0$

so $1-e^x < 0$ for all $x>0$.

Thus $1-e^x \to 0^-$ as $x \to 0^+$.

(b) $\lim\limits_{x\to 5} \frac{x^2-25}{x^2-3x-10}$

numerator $\to 0$, denominator $\to 0$

$$= \lim_{x\to 5} \frac{(x-5)(x+5)}{(x-5)(x+2)}$$

$$= \lim_{x\to 5} \frac{x+5}{x+2}$$

$$= \frac{10}{7}$$

(c) $\lim\limits_{x\to\infty} \frac{6x^2+5}{1+3x^2}$

numerator $\to \infty$, denominator $\to \infty$

$$= \lim_{x\to\infty} \frac{(6x^2+5)\left(\frac{1}{x^2}\right)}{(1+3x^2)\left(\frac{1}{x^2}\right)}$$

$$= \lim_{x\to\infty} \frac{6+5/x^2}{1/x^2+3}$$

$$= \frac{6}{3}$$

$$= 2$$

3. (2 points) A function f satisfies the following inequality for all $x \neq 0$.

$$\frac{3x + 2\sin x}{2x} \leq f(x) \leq \frac{7x - 2\sin x}{2x}$$

Determine $\lim_{x \to 0} (f(x))$.

$$\lim_{x \to 0} \frac{3x + 2\sin x}{2x} = \lim_{x \to 0} \left(\frac{3x}{2x} + \frac{2\sin x}{2x} \right)$$

$$= \lim_{x \to 0} \left(\frac{3}{2} + \frac{\sin x}{x} \right)$$

$$= \lim_{x \to 0} \frac{3}{2} + \lim_{x \to 0} \frac{\sin x}{x}$$

$$= \frac{3}{2} + 1 = \frac{5}{2}$$

$$\lim_{x \to 0} \frac{7x - 2\sin x}{2x} = \lim_{x \to 0} \left(\frac{7x}{2x} - \frac{2\sin x}{2x} \right)$$

$$= \lim_{x \to 0} \left(\frac{7}{2} - \frac{\sin x}{x} \right)$$

$$= \lim_{x \to 0} \left(\frac{7}{2} \right) - \lim_{x \to 0} \frac{\sin x}{x}$$

$$= \frac{7}{2} - 1 = \frac{5}{2}$$

By the Squeeze Theorem,

$$\lim_{x \to 0} f(x) = \frac{5}{2}$$