Math 220 ~~Quiz 2 (take-home)~~ Fall 2010

SOLUTIONS

Your Name ______________________________

TA's Name ______________________________

Discussion Section ______________________________
(list either section number or meeting times)

- You may work with other students in this class. However each student should write up solutions separately and independently – nobody should copy someone else's work.
- You may use your notes or the textbook.
- No calculators are allowed on any problem except #1
- You must show sufficient work to justify each answer.
- The quiz should be turned in to your TA by 4pm Thursday. You are allowed to turn it in early when you next see your TA. Although most TA's mailboxes are in 250 Altgeld your TA should inform you about the best way to submit your quiz.
- Be sure that the pages are nicely stapled – do not just fold the corners.
- Note to TA's – you should not help students with these specific problems or go over solutions until after 4pm Thursday.

1. (3 points) Let $P(t)$ represent the population of rabbits in a prairie t months since observation began. Fill in the missing entries $P(1)$, $P(2)$ and $P(3)$ given that the population grows exponentially. You may use a calculator for this problem, but you must still show sufficient work to justify your answers.

t	$P(t)$
0	200
1	299.1
2	447.2
3	668.7
4	1000

$$P(t) = C(a)^t$$
$$200 = C(a)^0 \Rightarrow C = 200$$
$$P(t) = 200(a)^t$$
$$1000 = 200(a)^4 \Rightarrow a = \sqrt[4]{5}$$
$$\approx 1.4953$$

$$P(t) = 200\left(\sqrt[4]{5}\right)^t = 200(5)^{t/4}$$

OR $P(t) \approx 200(1.4953)^t$

$$P(1) \approx 299.1$$
$$P(2) \approx 447.2$$
$$P(3) \approx 668.7$$

2. (2 points) Determine the exact value for each solution to the equation below.

$$\ln(4-x) + \ln(4+x) = 0$$

$$\ln((4-x)(4+x)) = 0$$
$$\ln(16-x^2) = 0$$
$$16 - x^2 = e^0$$
$$16 - x^2 = 1$$
$$15 = x^2$$
$$x = \pm\sqrt{15}$$

NOTE SINCE $\sqrt{15} \approx 3.9$, BOTH $\sqrt{15}$ AND $-\sqrt{15}$ ARE IN THE DOMAIN OF $\ln(4-x) + \ln(4+x)$ SO BOTH SOLUTIONS ARE ACCEPTABLE.

3. (2 points) Find the domain of the function $f(x) = \dfrac{e^{2x}+9}{e^{2x}-100}$

the denominator equals $0 \Leftrightarrow$

$e^{2x}-100=0 \Leftrightarrow e^{2x}=100 \Leftrightarrow \ln(e^{2x})=\ln(100)$

$\Leftrightarrow 2x=\ln(100) \Leftrightarrow x=\ln(100)/2$

since $\dfrac{\ln(100)}{2}=\dfrac{\ln(10^2)}{2}=\dfrac{2\ln(10)}{2}=\ln(10)$,

the domain of f is

$$(-\infty,\ln(10))\cup(\ln(10),\infty)$$

4. (2 points) Given that $f(x) = (\ln(2x-5))^3$, find a formula for $f^{-1}(x)$.

LET $y=f^{-1}(x)$

so $f(y)=x$

$(\ln(2y-5))^3=x$

$\ln(2y-5)=x^{1/3}$

$2y-5=e^{x^{1/3}}$

$2y=5+e^{x^{1/3}}$

$y=(5+e^{x^{1/3}})/2$

$$f^{-1}(x)=\frac{5+e^{x^{1/3}}}{2}=\frac{5+e^{\sqrt[3]{x}}}{2}$$

5. (1 point) Given that f is a one-to-one function determine the value of $f^{-1}(5)$ given that

$$f(-5)=-10,\ f(-3)=-5,\ f(3)=5 \text{ and } f(5)=10$$

$f(3)=5$ so $f^{-1}(5)=3$

the roles of x & y are switched