Math 220 (section ???) Quiz 13 Fall 2010

Name SOLUTIONS

You have 15 minutes for this quiz – no calculators allowed.

1. (4 points) Determine the average value of the function $f(x) = 5\sin\left(\frac{x}{2}\right)$ on the interval $[0, 2\pi]$. Simplify your answer as much as possible.

$$f_{ave} = \frac{1}{2\pi - 0}\int_0^{2\pi} 5\sin\left(\frac{x}{2}\right)dx$$

Let $u = \frac{x}{2}$

$du = \frac{1}{2}dx$

$2du = dx$

$x = 0 \Rightarrow u = 0$

$x = 2\pi \Rightarrow u = \pi$

$$= \frac{1}{2\pi}\int_0^{\pi} 5\sin(u)\,2\,du$$

$$= \frac{5}{\pi}\int_0^{\pi}\sin(u)\,du$$

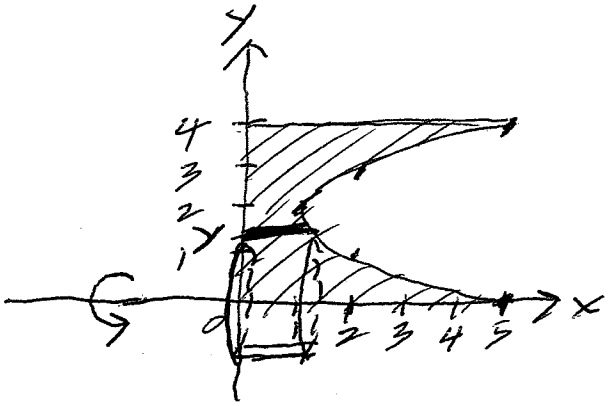
$$= \frac{5}{\pi}\left[-\cos(u)\right]_0^{\pi}$$

$$= \frac{5}{\pi}\left[-\cos(\pi) - (-\cos(0))\right]$$

$$= \frac{5}{\pi}\left[-(-1) + 1\right]$$

$$= \frac{10}{\pi}$$

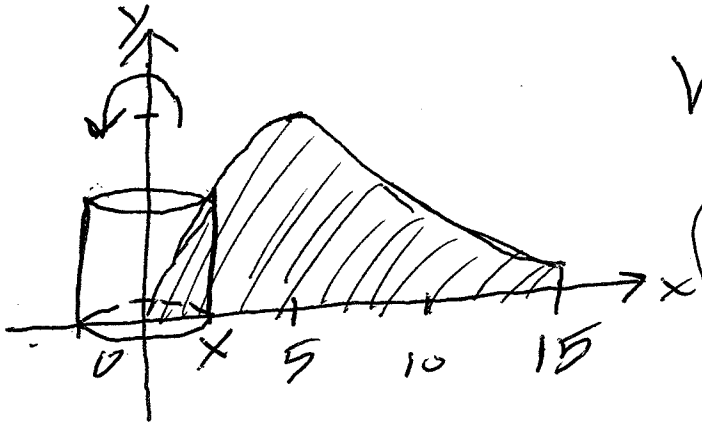
2. (3 points) Let **R** be the region bounded by the graphs of $x = (y-2)^2 + 1$, $x = 0$, $y = 0$, and $y = 4$. Set up, but do not evaluate, a definite integral which represents the volume of the solid obtained when **R** is revolved around the x-axis.



$$V = \int_0^4 2\pi (\text{radius})(\text{height})\, dy$$

$$V = \int_0^4 2\pi y ((y-2)^2 + 1)\, dy$$

3. (3 points) Let **R** be the region bounded by the graph of $y = 3xe^{-0.02x^2}$ and the x-axis on the interval $[0, 15]$. Set up, but do not evaluate, a definite integral which represents the volume of the solid obtained when **R** is revolved around the y-axis.



$$V = \int_0^{15} 2\pi (\text{radius})(\text{height})\, dx$$

$$V = \int_0^{15} 2\pi x \left(3xe^{-0.02x^2}\right) dx$$