Name: SOLUTIONS

You have 15 minutes for this quiz – no calculators allowed.

1. (2 points) Precisely state Rolle's Theorem.
   Let \( f \) be a function that satisfies the following three hypotheses:
   \( f \) is continuous on the closed interval \([a, b]\).
   \( f \) is differentiable on the open interval \((a, b)\).
   \( f(a) = f(b) \)
   Then there is a number \( c \) in \((a, b)\) such that \( f'(c) = 0 \).

2. (2 points) Let \( f(x) = 3x^2 + 2x + 5 \). Determine the value of \( c \) which satisfies the conclusion of the Mean Value Theorem on the interval \([a, b]\) with \( a < b \). You must show all work and simplify your answer.
   Polynomials are continuous and differentiable everywhere so \( f \) is continuous on \([a, b]\) and differentiable on \((a, b)\). The Mean Value Theorem implies there is a \( c \) in \((a, b)\) such that \( f'(c) = \frac{f(b) - f(a)}{b - a} \)
   Since \( f'(x) = 6x + 2 \) we obtain
   \[ 6c + 2 = \frac{(3b^2 + 2b + 5) - (3a^2 + 2a + 5)}{b - a} \]
   \[ 6c + 2 = \frac{3(b^2 - a^2) + 2(b - a)}{b - a} \]
   \[ 6c + 2 = \frac{3(b - a)(b + a) + 2(b - a)}{b - a} \]
   \[ 6c + 2 = (b - a)(3b + a + 2) / (b - a) \]
   \[ 6c + 2 = 3(b + a) + 2 \]
   \[ 6c = 3(b + a) \quad \rightarrow \quad c = \frac{b + a}{2} \]
3. (2 points each) Let \( R \) be the region bounded by the \( x \)-axis and the graph of \( y = 3 + 2 \sin x \) on the interval \([0, 2\pi]\). Set up, but do not evaluate, definite integrals which represent the given quantities. Use proper notation.

(a) The volume of the solid obtained when \( R \) is revolved around the \( x \)-axis.

\[
V = \int_0^{2\pi} \pi \left( 3 + 2 \sin x \right)^2 \, dx
\]

(b) The volume of the solid obtained when \( R \) is revolved around the line \( y = 7 \).

\[
V = \int_0^{2\pi} \pi \left( 7^2 - \pi \left( 7 - (3 + 2 \sin x) \right)^2 \right) \, dx
\]

(c) The volume of the solid with base \( R \) for which the cross-sections perpendicular to the \( x \)-axis are semi-circles.

\[
V = \int_0^{2\pi} \frac{1}{2} \pi \left( \text{radius} \right)^2 \, dx
\]

\[
= \int_0^{2\pi} \frac{1}{2} \pi \left( \frac{1}{2} \left( 3 + 2 \sin x \right) \right)^2 \, dx
\]