Math 220 (section ???) Quiz 12 Fall 2010

Name SOLUTIONS

You have 15 minutes for this quiz – no calculators allowed.

1. (2 points) Precisely state *Rolle's Theorem.*

Let f be a function that satisfies the following three hypotheses:

① f is continuous on the closed interval $[a,b]$.

② f is differentiable on the open interval (a,b).

③ $f(a) = f(b)$

Then there is a number c in (a,b) such that $f'(c) = 0$.

2. (2 points) Let $f(x) = 3x^2 + 2x + 5$. Determine the value of c which satisfies the conclusion of the *Mean Value Theorem* on the interval $[a, b]$ with $a < b$. You must show all work and simplify your answer.

Polynomials are continuous and differentiable everywhere so f is continuous on $[a,b]$ and differentiable on (a,b). The Mean Value Theorem implies there is a c in (a,b) such that $f'(c) = (f(b)-f(a))/(b-a)$

Since $f'(x) = 6x+2$ we obtain

$$6c+2 = ((3b^2+2b+5)-(3a^2+2a+5))/(b-a)$$

$$6c+2 = (3(b^2-a^2)+2(b-a))/(b-a)$$

$$6c+2 = (3(b-a)(b+a)+2(b-a))/(b-a)$$

$$6c+2 = (b-a)(3(b+a)+2)/(b-a)$$

$$6c+2 = 3(b+a)+2$$

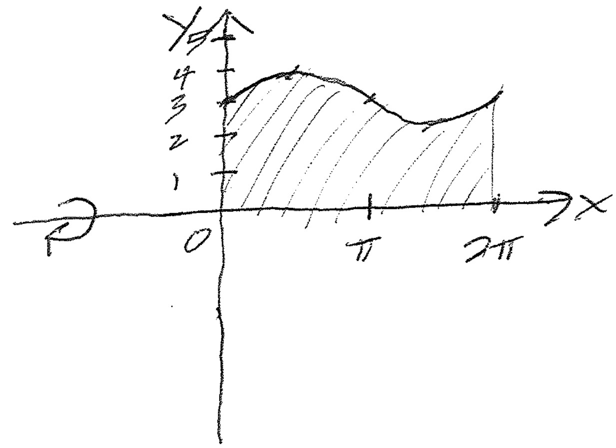
$$6c = 3(b+a) \longrightarrow c = \frac{b+a}{2}$$

is result true for all polynomials of degree 2?

3. (2 points each) Let **R** be the region bounded by the x-axis and the graph of $y = 3 + 2\sin x$ on the interval $[0, 2\pi]$. Set up, but do not evaluate, definite integrals which represent the given quantities. Use proper notation.

(a) The volume of the solid obtained when **R** is revolved around the x-axis.

$$V = \int_0^{2\pi} \pi (3 + 2\sin x)^2 \, dx$$

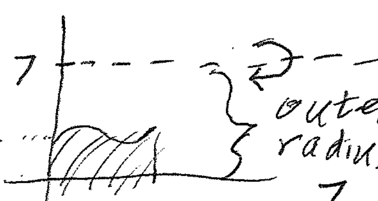


(b) The volume of the solid obtained when **R** is revolved around the line $y = 7$.

$$V = \int_0^{2\pi} \left(\pi (7)^2 - \pi (7 - (3 + 2\sin x))^2\right) dx$$

inner radius $7 - y$

or $7 - (3 + 2\sin x)$



outer radius 7

(c) The volume of the solid with base **R** for which the cross-sections perpendicular to the x-axis are semi-circles.

$$V = \int_0^{2\pi} \frac{1}{2}\pi (\text{radius})^2 \, dx$$

$$= \int_0^{2\pi} \frac{1}{2}\pi \left(\frac{1}{2}(3 + 2\sin x)\right)^2 dx$$

diameter is y

so radius is $\frac{1}{2}y = \frac{1}{2}(3 + 2\sin x)$