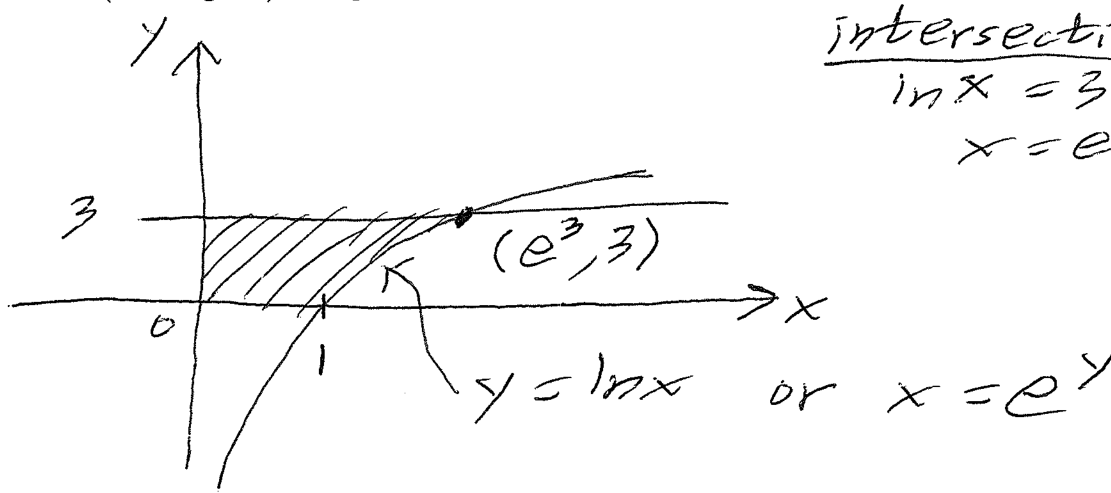
Math 220 (section ???) Quiz 11 Fall 2010

Name SOLUTIONS

You have 20 minutes for this quiz – no calculators allowed.

1. (3 points) The area of the first quadrant region bounded by the x-axis, the y-axis, the line $y = 3$ and the graph of $y = \ln x$ can be determined through the use of definite integrals. Show how to do this by writing the appropriate definite integral (or integrals) to represent this area. You do not need to evaluate any integrals.



intersection
$\ln x = 3$
$x = e^3$

$$\text{AREA} = \int_0^3 e^y \, dy$$

OR

$$\text{AREA} = \int_0^1 3 \, dx + \int_1^{e^3} (3 - \ln x) \, dx$$

integrating with respect to y is seen to be easier to set up and it would be easier to evaluate.

2. (3 points) Evaluate the definite integral.

$$\int_0^2 \frac{8x}{\sqrt{x^2+6}}\,dx = \int_6^{10} \frac{4\,du}{\sqrt{u}} = \int_6^{10} 4u^{-1/2}\,du$$

let $u = x^2+6$

$\frac{du}{dx} = 2x$

$du = 2x\,dx$

$4\,du = 8x\,dx$

when $x=0$, $u = 0^2+6 = 6$

when $x=2$, $u = 2^2+6 = 10$

$$= 4 \cdot \frac{1}{1/2} u^{1/2} \Big|_6^{10}$$

$$= 8\sqrt{u} \Big|_6^{10}$$

$$= 8\sqrt{10} - 8\sqrt{6}$$

3. (2 points) Evaluate the indefinite integral. Hint: try the substitution $u = x^3 + 4$.

$$\int \frac{3x^8}{x^3+4}\,dx = \int \frac{(x^3)^2 \cdot 3x^2\,dx}{x^3+4}$$

$u = x^3+4$

$\frac{du}{dx} = 3x^2$

$du = 3x^2\,dx$

note $x^3 = u-4$

$$= \int \frac{(u-4)^2\,du}{u}$$

$$= \int \frac{u^2-8u+16}{u}\,du$$

$$= \int \left(u - 8 + \frac{16}{u}\right) du$$

$$= \frac{1}{2}u^2 - 8u + 16\ln|u| + C$$

$$= \frac{1}{2}(x^3+4)^2 - 8(x^3+4) + 16\ln|x^3+4| + C$$

4. (1 point each) Suppose that f is an odd function and g is an even function which are each integrable on the interval $[-3,3]$. Given that $\int_0^3 f(x)\,dx = 4$ and $\int_0^3 g(x)\,dx = 5$, evaluate the following definite integrals.

(a) $\int_{-3}^{3} (6f(x) + 8g(x))\,dx$ $= 6\int_{-3}^{3} f(x)\,dx + 8\int_{-3}^{3} g(x)\,dx$

$= 6\cdot 0 + 8\cdot 2\int_0^3 g(x)\,dx$

$= 0 + 16\int_0^3 g(x)\,dx$

$= 16(5)$

$= 80$

notes:
$\int_{-3}^{3} f(x)\,dx = 0$
since f is odd

$\int_{-3}^{3} g(x)\,dx =$
$2\int_0^3 g(x)\,dx$
since g is even

(b) $\int_{-3}^{3} \left(5 + 4\left(f(x)\right)^7\right)\,dx$ $= \int_{-3}^{3} 5\,dx + 4\int_{-3}^{3} (f(x))^7\,dx$

$= 30 + 4\cdot 0$

$= 30$

notes:

f is odd implies
$f(-x) = -f(x)$
now
$(f(-x))^7 = (-f(x))^7$
$= -(f(x))^7$
so $(f(x))^7$ is
also odd
Thus $\int_{-3}^{3} (f(x))^7\,dx = 0$

notes:
$\int_{-3}^{3} 5\,dx$ represents this area which is 30