Math 220 (section ???) Quiz 10 Fall 2010

Name SOLUTIONS

You have 20 minutes for this quiz – no calculators allowed.

1. (2 points) Evaluate the following indefinite integral.

$$\int \frac{6}{x^4}\,dx = \int 6x^{-4}\,dx = 6\left(\frac{1}{-3}x^{-3}\right) + C$$

$$= -2x^{-3} + C$$

$$\text{or } \frac{-2}{x^3} + C$$

2. (2 points) Evaluate and simplify the following definite integral.

$$\int_2^{18} \frac{20x+1}{2x}\,dx = \int_2^{18}\left(\frac{20x}{2x} + \frac{1}{2x}\right)dx$$

$$= \int_2^{18}\left(10 + \frac{1}{2}\cdot\frac{1}{x}\right)dx$$

$$= \left(10x + \frac{1}{2}\ln|x|\right)\Big|_2^{18}$$

$$= \left(180 + \frac{1}{2}\ln(18)\right) - \left(20 + \frac{1}{2}\ln(2)\right)$$

$$= 160 + \frac{1}{2}\left(\ln(18) - \ln(2)\right)$$

$$= 160 + \frac{1}{2}\ln\left(\frac{18}{2}\right)$$

$$= 160 + \frac{1}{2}\ln(9)$$

$$= 160 + \frac{1}{2}\ln(3^2)$$

$$= 160 + \ln(3)$$

3. (2 points) Evaluate the following indefinite integral.

$$\int \frac{4\cos x - 4\cos^3 x}{\sin^2 x}\,dx = \int \frac{4\cos x\,(1-\cos^2 x)}{\sin^2 x}\,dx$$

$$= \int \frac{4\cos x\,(\sin^2 x)}{\sin^2 x}\,dx$$

$$= \int 4\cos x\,dx$$

$$= 4\sin x + C$$

4. (2 points) Rounded off to one place after the decimal, $\int_{30}^{35} \left(\cos^4(3x+8)+4\right)\,dx$ is equal to one of the choices below. Circle the correct choice and show enough work to justify your answer. Hint: Obtain an approximation without finding an antiderivative or a limit.

(a) 3.4 (b) 9.3 (c) 13.7 (d) 18.9 (e) 21.8

(f) 26.5 (g) 32.1 (h) 35.0 (i) 41.6 (j) 47.2

$-1 \le \cos(3x+8) \le 1$

$0 \le \cos^4(3x+8) \le 1$

$4 \le \cos^4(3x+8)+4 \le 5$

$\int_{30}^{35} 4\,dx \le \int_{30}^{35} (\cos^4(3x+8)+4)\,dx \le \int_{30}^{35} 5\,dx$

$20 \le \int_{30}^{35} (\cos^4(3x+8)+4)\,dx \le 25$

see property 8 in section 5.2

5. (2 points) Evaluate the following limit. Be sure to use proper notation throughout your evaluation of this limit.

$$\lim_{n\to\infty}\sum_{k=1}^{n}\left(\frac{5}{n}+\frac{8k}{n^2}+\frac{100}{n^3}\right)=\lim_{n\to\infty}\left(\sum_{k=1}^{n}\frac{5}{n}+\sum_{k=1}^{n}\frac{8k}{n^2}+\sum_{k=1}^{n}\frac{100}{n^3}\right)$$

$$=\lim_{n\to\infty}\left(\frac{5}{n}\sum_{k=1}^{n}1+\frac{8}{n^2}\sum_{k=1}^{n}k+\frac{100}{n^3}\sum_{k=1}^{n}1\right)$$

$$=\lim_{n\to\infty}\left(\frac{5}{n}\cdot n+\frac{8}{n^2}\cdot\frac{n(n+1)}{2}+\frac{100}{n^3}\cdot n\right)$$

$$=\lim_{n\to\infty}\left(5+\frac{8n^2+8n}{2n^2}+\frac{100n}{n^3}\right)$$

$$=5+\frac{8}{2}+0$$

$$=9$$