MATH 220 Test 2 Fall 2010

Name SOLUTIONS

(circle your TA discussion section)

- ▷ **BD1**, MW 11:00-12:50, Noel DeJarnette
- ▷ **BD2**, TR 9:00-9:50, Kyle Knee
- ▷ **BD3**, TR 10:00-10:50, Scott Tichenor
- ▷ **BD4**, TR 2:00-2:50, Dan Schultz
- ▷ **BD5**, TR 11:00-11:50, Scott Tichenor
- ▷ **BD7**, MW 1:00-2:50, Scott Wetter
- ▷ **BD8**, TR 12:00-12:50, Hong Liu
- ▷ **BD9**, TR 1:00-1:50, Dan Schultz
- ▷ **DD1**, TR 11:00-11:50, Sarah Son
- ▷ **DD2**, TR 10:00-10:50, Sarah Son
- ▷ **DD4**, TR 12:00-12:50, Donghoon Jang
- ▷ **DD5**, TR 1:00-1:50, Donghoon Jang
- ▷ **DD6**, MW 1:00-2:50, Jesse Beder
- ▷ **DD7**, TR 8:00-8:50, Mu-Tsun Tsai
- ▷ **DD8**, MW 9:00-10:50, Joseph Vandehey
- ▷ **DD9**, TR 9:00-9:50, Mu-Tsun Tsai
- ▷ **AD1**, TR 11:00-11:50, Stephen Longfield
- ▷ **AD2**, TR 2:00-2:50, Lisa Hickok
- ▷ **AD3**, TR 1:00-1:50, Stephen Longfield
- ▷ **AD6**, TR 9:00-9:50, Lisa Hickok
- ▷ **AD7**, TR 3:00-3:50, Sarah Yeakel
- ▷ **AD8**, TR 4:00-4:50, Daniel Hockensmith

- Sit in your assigned seat (shown below).
- Do not open this test booklet until I say *START*.
- Turn off all electronic devices and put away all items except a pen/pencil and an eraser.
- You must show sufficient work to justify each answer.
- While the test is in progress, we will not answer questions concerning the test material.
- Quit working and close this test booklet when I say *STOP*.
- Quickly turn in your test to me or a TA and show your Student ID.

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FRONT OF ROOM – 314 Altgeld Hall

1. (5 points) Evaluate the following derivatives.

(a) $\frac{d}{dx}(\cos x) =$ $-\sin x$

(b) $\frac{d}{dx}(\csc x) =$ $-\csc x \cot x$

(c) $\frac{d}{dx}(\tan x) =$ $\sec^2 x$

(d) $\frac{d}{dx}\left(\sin^{-1} x\right) =$ $\frac{1}{\sqrt{1-x^2}}$

(e) $\frac{d}{dx}(\ln x) =$ $\frac{1}{x}$

2. (8 points) Find $g'(t)$ given that $g(t) = 5t^5 + \sqrt{t} - 40$ $= 5t^5 + t^{1/2} - 40$

$$g'(t) = 5(5t^4) + \frac{1}{2}t^{-1/2}$$

$$g'(t) = 25t^4 + \frac{1}{2\sqrt{t}}$$

3. (8 points) Find $f'(x)$ given that $f(x) = \frac{x^5}{\sin x}$

$$f'(x) = \frac{(x^5)'(\sin x) - (x^5)(\sin x)'}{(\sin x)^2}$$

$$f'(x) = \frac{5x^4 \sin x - x^5 \cos x}{\sin^2 x}$$

4. (8 points) Find $P'(t)$ given that $P(t) = \left(t^9 - 10t^4 + 12\right)^8$

$$P'(t) = 8(t^9 - 10t^4 + 12)^7 \cdot (9t^8 - 40t^3)$$

5. (5 points) Find $\frac{dy}{dx}$ given that $x^5e^y = 2x^3 + 5y^2 + 6$. It is okay to leave your answer in terms of both x and y.

$$\frac{d}{dx}(x^5e^y) = \frac{d}{dx}(2x^3+5y^2+6)$$

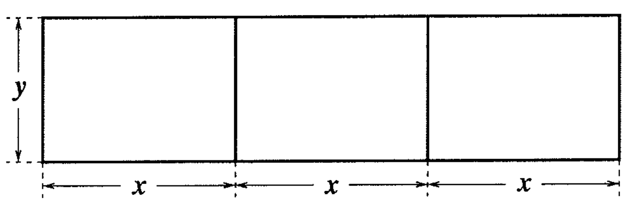
$$5x^4e^y + x^5e^y\frac{dy}{dx} = 6x^2 + 10y\frac{dy}{dx}$$

$$x^5e^y\frac{dy}{dx} - 10y\frac{dy}{dx} = 6x^2 - 5x^4e^y$$

$$\frac{dy}{dx}(x^5e^y - 10y) = 6x^2 - 5x^4e^y$$

$$\frac{dy}{dx} = \frac{6x^2 - 5x^4e^y}{x^5e^y - 10y}$$

6. (10 points) A farmer wishes to fence off three identical adjoining rectangular pens as in the diagram shown, but he only has 600 feet of fencing available. Determine the values for x and y which will maximize the total area enclosed by these three pens.



AREA | $A = 3xy$

LENGTH | $600 = 6x + 4y$

$$y = \frac{600-6x}{4}$$

$$A = 3\cancel{x}y$$

$$A = 3x\left(\frac{600-6x}{4}\right)$$

$$A = 3x\left(150 - \tfrac{3}{2}x\right)$$

$$A = 450x - \tfrac{9}{2}x^2$$

maximize A for x in the interval $(0, 100)$ (do you see why?)

$A' = 450 - 9x$

values of A'

$+ + \; 0 \; - -$

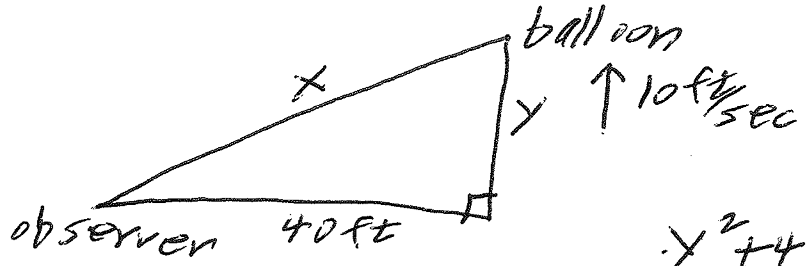
$0 \quad 50 \quad 100 \quad x$

max. area at

$x = 50$

$y = \frac{600-6(50)}{4} = 75$

7. (10 points) A small balloon is released at a point 40 feet away from an observer, who is on level ground. If the balloon goes straight up at a rate of 10 feet per second, how fast is the distance from the observer to the balloon increasing when the balloon is 30 feet high?



given $\frac{dy}{dt} = 10 \text{ ft/sec}$

want $\left.\frac{dx}{dt}\right|_{y=30\text{ft}}$

$y^2 + 40^2 = x^2$

$2y\frac{dy}{dt} + 0 = 2x\frac{dx}{dt}$

$y\frac{dy}{dt} = x\frac{dx}{dt}$

From Pythagorus, $x = 50$ when $y = 30$. Thus

$(30)(10) = 50\frac{dx}{dt}$

and $\frac{dx}{dt} = \frac{300}{50} = 6 \text{ ft/sec}$

8. (10 points) What are the coordinates (x, y) for the highest point on the graph of the function $g(x) = 180x - 10e^{2x}$?

$g'(x) = 180 - 10e^{2x} \cdot 2$

$g'(x) = 180 - 20e^{2x}$

setting $g'(x) = 0$ gives

$0 = 180 - 20e^{2x}$

$20e^{2x} = 180$

$e^{2x} = 9$

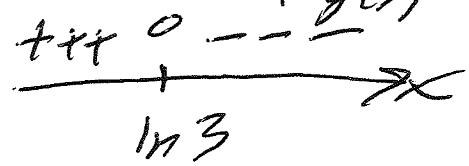
$\ln(e^{2x}) = \ln(9)$

$2x = \ln(9)$

$x = \frac{1}{2}\ln(9)$

$x = \frac{1}{2}\ln(3^2) = \ln 3$

values of $g'(x)$



abs. max. when $x = \ln 3$

$(x, y) = (\ln 3, 180\ln 3 - 10e^{2\ln 3})$

$(x, y) = (\ln 3, 180\ln 3 - 90)$

9. (6 points each) A function $f(x)$ has first derivative $f'(x) = e^{0.5x}(10x - 60)$.

(a) Upon which interval is $f(x)$ increasing?

values of $f'(x)$

$- - - \; 0 \; + + +$

$\xrightarrow{\hspace{6em}} x$

6

f is increasing on the interval

$(6, \infty)$

answer is $[6, \infty)$ by book's orignal definition of increasing

(b) Upon which interval is the graph of $f(x)$ concave down?

$$f''(x) = (e^{0.5x})'(10x-60) + (e^{0.5x})(10x-60)'$$

$$= 0.5e^{0.5x}(10x-60) + e^{0.5x}(10)$$

$$= e^{0.5x}(0.5(10x-60) + 10)$$

$$= e^{0.5x}(5x-20)$$

values of $f''(x)$

$- - - \; 0 \; + + +$

$\xrightarrow{\hspace{6em}} x$

4

f is concave down on $(-\infty, 4)$

answer is $(-\infty, 4]$ by book's original definition of concave down

10. (6 points) Determine a formula for w as a function of s so that $\frac{dw}{ds} = 10s$ and $w(1) = 2$.

$w = 5s^2 + C$

since $w(1) = 2$, we get

$2 = 5(1)^2 + C$

$C = -3$

$w = 5s^2 - 3$

11. (6 points) Determine a formula for w as a function of s so that $\frac{dw}{ds} = 10w$ and $w(1) = 2$.

$w = Ce^{10s}$

since $w(1) = 2$, we get

$2 = Ce^{10(1)}$

$C = \frac{2}{e^{10}}$

$w = \left(\frac{2}{e^{10}}\right)e^{10s}$

or

$w = 2e^{10s-10}$

12. (6 points each) Evaluate the following limits.

(a) $\lim_{x\to 0^+} \frac{\ln(x^3+3x)}{\ln x}$ $\xrightarrow{}-\infty$, $\xrightarrow{}-\infty$

$$= \lim_{x\to 0^+} \frac{\left(\frac{3x^2+3}{x^3+3x}\right)}{\left(\frac{1}{x}\right)}$$ by L'Hospital's Rule

$$= \lim_{x\to 0^+} \frac{3x^3+3x}{x^3+3x} \quad \begin{matrix}\to 0\\ \to 0\end{matrix}$$

$$= \lim_{x\to 0^+} \frac{9x^2+3}{3x^2+3}$$ by L'Hospital's Rule

$$= \frac{9(0)^2+3}{3(0)^2+3}$$

$$= 1$$

(b) $\lim_{x\to\infty} x^{200}e^{-x}$

$$= \lim_{x\to\infty} \frac{x^{200}}{e^x}$$

$x^{200} \to \infty$ slowly

$e^x \to \infty$ quickly

$$= 0$$

another approach is to use L'Hospital's Rule repeatedly to get

$$\lim_{x\to\infty} \frac{x^{200}}{e^x} = \lim_{x\to\infty} \frac{200x^{199}}{e^x} = \lim_{x\to\infty} \frac{200\cdot 199 x^{198}}{e^x} =$$

$$\dots = \lim_{x\to\infty} \frac{200!}{e^x} = 0 \quad \text{since}$$

$200! = 200\cdot 199\cdot 198\cdots 3\cdot 2\cdot 1$ is a constant

Students – do not write on this page!

1 (5 points) ____________________

2 (8 points) ____________________

3 (8 points) ____________________

4 (8 points) ____________________

5 (5 points) ____________________

6 (10 points) ____________________

7 (10 points) ____________________

8 (10 points) ____________________

9a (6 points) ____________________

9b (6 points) ____________________

10 (6 points) ____________________

11 (6 points) ____________________

12a (6 points) ____________________

12b (6 points) ____________________

TOTAL (100 points) ____________