Name ______________________________

You have 12 minutes for this quiz.

1. (4 points) Approximate $\sqrt[3]{1018}$ using Newton's Method. You should determine a reasonable first approximation but use Newton's Method to determine a second approximation. Simplify this second approximation.

use $f(x) = x^3 - 1018$ so that $\sqrt[3]{1018}$ is a root of $f(x)$. $10^3 = 1000 \approx 1018$ so $x_0 = 10$ is a reasonable first guess.

then,

$$x_1 = x_0 - \frac{f(x_0)}{f'(x_0)}$$

$$= x_0 - \frac{x_0^3 - 1018}{3x_0^2}$$

$$= 10 - \frac{10^3 - 1018}{3 \cdot 10^2}$$

$$\frac{18}{3 \cdot 10^2}$$

$$= 10.06$$

2. (3 points) Students are asked to use a linear approximation (or differential) to estimate the value of $\sqrt[5]{e}$ without the use of a calculator. One student uses the function $x^{1/5}$ and another student uses the function e^x. Choose which function is more appropriate for this problem, find its linear approximation (or differential), and use this to estimate $\sqrt[5]{e}$.

$\sqrt[5]{e} = e^{0.2}$, and the value of $f(x) = e^x$ is known when

$x = 0$ which is close to 0.2, so

$$e^{.2} = f(0.2) \approx f(0) + f'(0)(0.2 - 0)$$

$$= 1 + 1(0.2)$$

$$= 1.2$$

$f'(x) = (e^x)' = e^x$

3. (3 points) The measured radius of a circle has a very small relative error. Which of the following is the approximate corresponding relative error in the area of the circle? You must show sufficient work to justify your answer.

(a) Three times the relative error in the radius.

(b) The square of the relative error in the radius.

(c) π times the relative error in the radius.

(d) Two times the relative error in the radius.

(e) The square root of the relative error in the radius.

$A = \pi r^2$

$dA = 2\pi r \, dr$

$$\frac{dA}{A} \approx \frac{2\pi r dr}{\pi r^2}$$

$$= 2 \frac{dr}{r}$$