Math 220 – Quiz 8

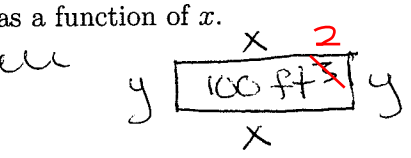
Name Solutions

You have 12 minutes for this quiz.

1. (4 points) Rudy wants to enclose a 100 square foot rectangular region to be used for a garden. He will use fencing which costs \$16 per foot along three sides, and fencing which costs \$34 per foot along the fourth side. Let x represent the length of that fourth side.

   (a) Find a formula for the total cost of all the fencing as a function of x.

   We have the diagram

   so $C = 16(x + 2y) + 34x$

   $= 50x + 32y.$

   Now $100 = xy$, so $y = \frac{100}{x}$, and therefore

   $C = 50x + \frac{3,200}{x}.$

   (b) Find the dimensions which give the minimum total cost.

   $0 = \frac{dC}{dx} = 50 - \frac{3,200}{x^2} \Rightarrow 50x^2 = 3,200$

   $\Rightarrow x^2 = 64 \Rightarrow x = 8$, so $y = \frac{100}{8} = \frac{25}{2}$, so

   the dimensions that minimize cost

   are 8 ft by 12.5 ft.

   (c) Find the minimum total cost.

   $C_{min} = 50(8) + \frac{3,200}{8} = 400 + 400 = \$800$

2. (4 points) Evaluate the following limits. Show sufficient work to justify your answer.

(a) $\lim_{x\to 0} \frac{e^{6x}-1-6x}{4x^2}$ $\overset{\text{"}}{=}$ $\frac{e^0-1}{0} = \frac{0}{0}$, so by L'Hôpital's rule,

$\lim_{x\to 0} \frac{e^{6x}-1-6x}{4x^2} = \lim_{x\to 0} \frac{6e^{6x}-6}{8x}$ $\overset{\text{"}}{=}$ $\frac{6e^0-6}{0} = \frac{0}{0}$, so again by L'Hôpital's rule,

$\lim_{x\to 0} \frac{e^{6x}-1-6x}{4x^2} = \lim_{x\to 0} \frac{6e^{6x}-6}{8x} = \lim_{x\to 0} \frac{36e^{6x}}{8}$

$= \frac{36}{8} = \frac{9}{2}$

(b) $\lim_{x\to\infty} x \sin\left(\frac{1}{x}\right) = \lim_{x\to\infty} \frac{\sin\left(\frac{1}{x}\right)}{\frac{1}{x}}$ $\overset{\text{"}}{=}$ $\frac{\sin 0}{0} = \frac{0}{0}$, so by L'Hôpital's rule,

$\lim_{x\to\infty} x \sin\left(\frac{1}{x}\right) = \lim_{x\to\infty} \frac{\sin\left(\frac{1}{x}\right)}{\frac{1}{x}} = \lim_{x\to\infty} \frac{-\frac{1}{x^2}\cos\left(\frac{1}{x}\right)}{-\frac{1}{x^2}}$

$= \lim_{x\to\infty} \cos\left(\frac{1}{x}\right) = \cos 0 = 1.$

3. (2 points) Evaluate the following limits. No work needs to be shown.

(a) $\lim_{x\to\infty} \frac{10e^x}{x^{250}} = \infty$ Because exponentials grow faster than polynomials

(b) $\lim_{x\to\infty} \frac{\ln x}{\sqrt[3]{x}} = 0$ Because powers of x grow faster than logarithms