Name ________________________________

You have 10 minutes for this quiz.

1. (3 points) Carefully write down the content of The Mean Value Theorem.

Let \( f \) be a function satisfying the following:

1. \( f \) is continuous on \([a, b]\)
2. \( f \) is differentiable on \((a, b)\)

Then there is a number \( c \) in \((a, b)\) such that

\[ f'(c) = \frac{f(b) - f(a)}{b - a} \]

2. (3 points) A function \( f \) has the following second derivative. Determine the \( x \)-value for each inflection point on the graph of \( f \).

\[ f''(x) = 4e^x(x-3)(x-5)^2 \]

Since \( 4e^x \) and \((x-5)^2\) are never negative and \((x-3)\) is negative when \( x < 3 \) and positive when \( x > 3 \), \( f''(x) = 4e^x(x-3)(x-5)^2 \) is negative for \( x < 3 \) and positive for \( x > 3 \) (except for \( x = 5 \), where \( f''(x) = 0 \)), so the only inflection point is at \( x = 3 \).
3. (4 points) Upon which interval is the graph of \( f(x) = 2 + 3xe^{-4x} \) increasing?

\[
f'(x) = 3e^{-4x} - 12xe^{-4x} = 3e^{-4x}(1-4x).
\]

Since \( 3e^{-4x} \) is always positive and \( 1-4x > 0 \iff x < \frac{1}{4} \), we have

\[
f'(x) = 3e^{-4x}(1-4x) > 0 \iff x < \frac{1}{4}.
\]

Thus \( f(x) \) is increasing on the interval \( (-\infty, \frac{1}{4}) \).