

Name \_\_\_\_\_

You have 10 minutes for this quiz.

1. (3 points) Carefully write down the content of
- The Mean Value Theorem*
- .

Let  $f$  be a function satisfying the following:

- ①  $f$  is continuous on  $[a, b]$   
 ②  $f$  is differentiable on  $(a, b)$

Then there is a number  $c$  in  $(a, b)$

such that 
$$f'(c) = \frac{f(b) - f(a)}{b - a}$$

2. (3 points) A function
- $f$
- has the following second derivative. Determine the
- $x$
- value for each inflection point on the graph of
- $f$
- .

$$f''(x) = 4e^x(x-3)(x-5)^2$$

Since  $4e^x$  and  $(x-5)^2$  are never negative and  $(x-3)$  is negative when  $x < 3$  and positive when  $x > 3$ ,  $f''(x) = 4e^x(x-3)(x-5)^2$  is negative for  $x < 3$  and positive for  $x > 3$  (except for  $x=5$ , where  $f''(x)=0$ ), so the only inflection point is at  $x=3$ .

3. (4 points) Upon which interval is the graph of  $f(x) = 2 + 3xe^{-4x}$  increasing?

$$f'(x) = 3e^{-4x} - 12xe^{-4x} = 3e^{-4x}(1-4x).$$

Since  $3e^{-4x}$  is always positive and

$$1-4x > 0 \iff x < \frac{1}{4} \quad \text{we have}$$

$$f'(x) = 3e^{-4x}(1-4x) > 0 \iff x < \frac{1}{4}.$$

Thus  $f(x)$  is increasing on the interval  $(-\infty, \frac{1}{4})$ .