Name ______________________________

- For this quiz you can use calculators, textbooks, other students, etc.
- Be sure that all pages of your quiz are stapled together.
- There is no specific time limit, but the quiz is due at the beginning of your first discussion section meeting next week.
- Note to TA's – you should not help students with these specific problems or go over solutions until next Wednesday.

1. (3 points) A man lives in a high-rise apartment building. He leans out from one of his apartment windows and throws a ball upward. Between the time that the ball is thrown and the time that the ball hits the ground, the height of the ball is given by the formula $h(t) = -16t^2 + 96t + 160$, where t is the number of seconds since the ball is first thrown and $h(t)$ is measured in feet above ground-level.

   (a) Determine a formula for the velocity of the ball at time t.

   $V(t) = h'(t) = \boxed{-32t + 96}$ #

   (b) What is the ball's velocity 2 seconds after reaching its maximum height?

   Maximum: $V(t) = 0 \Rightarrow -32t + 96 = 0 \Rightarrow t = 3.$

   $\therefore V(3+2) = V(5) = -32 \times 5 + 96 = \boxed{-64 \text{ ft/s}}$ #

   (c) What is the ball's velocity when it hits the ground?

   ground: $h(t) = 0 \Rightarrow -16t^2 + 96t + 160 = 0$

   $\Rightarrow t = 3 + \sqrt{19}$ or $3 - \sqrt{19}$.

   $3 - \sqrt{19} < 0$ so it is impossible.

   $\therefore t = 3 + \sqrt{19}$.

   $\therefore V(3+\sqrt{19}) = -32 \times (3 + \sqrt{19}) + 96 = \boxed{-32\sqrt{19} \text{ ft/s}}$

   #.

2. (3 points) Suppose that y is a function of x whose graph goes through the point $(\ln 9, 24)$. Further suppose that $\frac{dy}{dx} = 0.5y$. Determine a formula for y.

$$\frac{dy}{dx} = 0.5y \Rightarrow y = C \cdot e^{0.5x}.$$

Since $y(\ln 9) = 24$, we know

$$C \cdot e^{0.5 \times \ln 9} = 24 \Rightarrow C = 8.$$

$$\therefore \boxed{y = 8e^{0.5x}}$$

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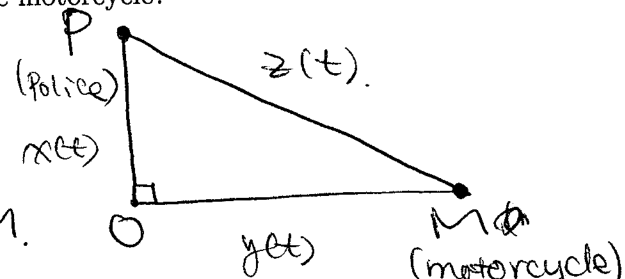
3. (4 points) A police car, approaching a right-angled intersection from the north, is chasing a speeding motorcycle that has turned a corner and is now moving straight east. When the police car is 0.6 miles north of the intersection and the motorcycle is 0.8 miles to the east, the police determine with radar that the distance between them and the motorcycle is increasing at 20 miles per hour. If the police car is moving at 60 miles per hour at the instant of measurement, then what is the speed of the motorcycle?



Let $x(t)$ be the length of PO, $y(t)$ be the length of OM, and $z(t)$ be the length of PM.

Let t_0 be the instant of measurement. Then we know by assumption, $x(t_0) = 0.6$, $y(t_0) = 0.8$, $\frac{dz}{dt}(t_0) = 20$, $\frac{dx}{dt}(t_0) = -60$. By Pythagorean theorem,

(*) $x^2(t) + y^2(t) = z^2(t)$, in particular, $x^2(t_0) + y^2(t_0) = z^2(t_0)$, so $z^2(t_0) = 1$. Differentiating (*) ~~and putting t=to~~,

we get $2x \cdot \frac{dx}{dt} + 2y \cdot \frac{dy}{dt} = 2z \frac{dz}{dt}$. Let $t = t_0$ and it becomes

$$2 \times 0.6 \times (-60) + 2 \times 0.8 \times \frac{dy}{dt}(t_0) = 2 \times 1 \times 20.$$

Solving for $\frac{dy}{dt}(t_0)$, we get $\frac{dy}{dt}(t_0) = \boxed{70 \text{ mph.}}$

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