Math 220 (section ?) – Quiz 5

Name _____

You have 10 minutes for this quiz.

1. (4 points) Find $\dfrac{dy}{dx}$ given that $\sec y = y^3x^5$.

$$\begin{aligned} \frac{d\sec y}{dx} &= \frac{d(y^3x^5)}{dx} \\ \sec y\tan y\frac{dy}{dx} &= 3y^2x^5\frac{dy}{dx} + 5y^3x^4 \\ (\sec y\tan y - 3y^2x^5)\frac{dy}{dx} &= 5y^3x^4 \\ \frac{dy}{dx} &= \frac{5y^3x^4}{\sec y\tan y - 3y^2x^5} \end{aligned}$$

2. (3 points each) Find derivatives of the following functions.

   (a) $y = \sin^{-1}\left(t^3+5\right)$

$$\begin{aligned} \sin y &= t^3+5 \\ \cos y\frac{dy}{dt} &= 3t^2 \\ \frac{dy}{dt} &= \frac{3t^2}{\cos y} \\ &= \frac{3t^2}{\sqrt{1-\sin^2 y}} \text{ because } \cos y \text{ is positive} \\ &= \frac{3t^2}{\sqrt{1-(t^3+5)^2}} \end{aligned}$$

(b) $y = \dfrac{(2x^3+5)^2 (5x+1)^3}{\sqrt{x^2+1}}$

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\begin{aligned}
\ln y &= 2\ln(2x^3+5) + 3\ln(5x+1) - \frac{1}{2}\ln(x^2+1) \\
\frac{d\ln y}{dx} &= \frac{12x^2}{2x^3+5} + \frac{15}{5x+1} - \frac{x}{x^2+1} \\
\frac{dy/dx}{y} &= \frac{12x^2}{2x^3+5} + \frac{15}{5x+1} - \frac{x}{x^2+1} \\
\frac{dy}{dx} &= y \times \left(\frac{12x^2}{2x^3+5} + \frac{15}{5x+1} - \frac{x}{x^2+1} \right) \\
&= \frac{(2x^3+5)^2 (5x+1)^3}{\sqrt{x^2+1}} \left(\frac{12x^2}{2x^3+5} + \frac{15}{5x+1} - \frac{x}{x^2+1} \right)
\end{aligned}
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