Name ____________________________________

1. (4 points) Let $f(x) = 4x^3 - 5$. Use the definition of a derivative as a limit to show that $f'(x) = 12x^2$. Show each step in your calculation and be sure to use proper terminology.

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$$= \lim_{h \to 0} \frac{4(x+h)^3 - 5 - (4x^3 - 5)}{h}$$

$$= \lim_{h \to 0} \frac{4(x^3 + 3x^2h + 3xh^2 + h^3) - 5 - 4x^3 + 5}{h}$$

$$= \lim_{h \to 0} \frac{12x^2h + 12xh^2 + 4h^3}{h}$$

$$= \lim_{h \to 0} \frac{h(12x^2 + 12xh + 4h^2)}{h}$$

$$= \lim_{h \to 0} (12x^2 + 12xh + 4h^2)$$

$$= 12x^2$$

2. (3 points) Find the equation of the line tangent to the graph of $f(x) = 3x^2 + 2x + 4$ at the point $(1, 9)$. Write your answer in the form $y = mx + b$. You may use any of the short-cut approaches discussed in section 3.1.

$m = f'(x) = 6x + 2$

$m = f'(1) = 6 \times 1 + 2 = 8$

point $(x_1, y_1) = (1, 9)$

$y - y_1 = m(x - x_1)$

$y - 9 = 8(x - 1) \Rightarrow \boxed{y = 8x + 1}$

3. (3 points) Given the graph of $f(x)$ shown below, sketch a graph of $f'(x)$.