

Name _____

TA/Section _____

- You can use calculators, textbooks, notes, other students, etc.
- This quiz is due by 4:00PM Friday in your TA's mailbox in 250 Altgeld Hall.

1. (2 points) A spring has a natural length of 2 m. A force of 24 N stretches the spring to a length of 2.4 m. Starting at its natural length, how much work will it take to stretch the spring to a length of 2.6 m?

$$F = kx$$

$$k = \frac{24}{2.4-2} = 60 \text{ N/m}$$

$$\int_0^{2.6-2} 60x \, dx = 30x^2 \Big|_0^{0.6} = 10.8 \text{ J}$$

2. (2 points) Find the average value of the function $f(x) = 5 \sin\left(\frac{x}{2}\right)$ on the interval $[0, 2\pi]$.

$$\frac{1}{2\pi} \int_0^{2\pi} 5 \sin\left(\frac{x}{2}\right) dx$$

$$= \frac{5}{2\pi} \int_0^{2\pi} \sin\left(\frac{x}{2}\right) dx$$

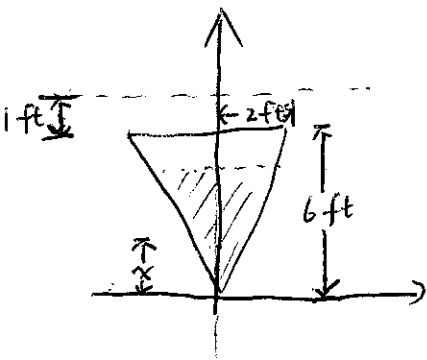
$$= \frac{5}{\pi} \left[-\cos\left(\frac{x}{2}\right) \right] \Big|_0^{2\pi}$$

$$= \frac{5}{\pi} [1 - (-1)] = \frac{10}{\pi}$$

3. (3 points) Let R be the region bounded by the x -axis and the graph of $y = x^3 - 9x$ on the interval $[0, 3]$. Set up, but do not evaluate, a definite integral for the volume of the solid obtained when R is revolved around the line $x = 5$.

$$\int_0^3 2\pi (5-x) (-x^3 + 9x) dx$$

4. (3 points) An inverted conical tank has a 2 foot radius at the top and is 6 feet high. It is filled to a height of 5 feet with olive oil weighing 57 lb/ft^3 . Set up, do not evaluate, a definite integral which represents the amount of work that it takes to pump the oil to a point 1 foot above the top rim of the tank.



$$F = 57 \cdot \frac{1}{3} \pi \left(\frac{x}{3}\right)^2 \Delta x \cdot 32$$

$$\int_0^5 32 \cdot 57 \cdot \frac{1}{3} \pi \left(\frac{x}{3}\right)^2 (7-x) dx$$

$$= \frac{57 \cdot 1804 \pi}{27} \int_0^5 x^2 (7-x) dx$$

$$= \frac{19808}{3} \pi \int_0^5 x^2 (7-x) dx$$