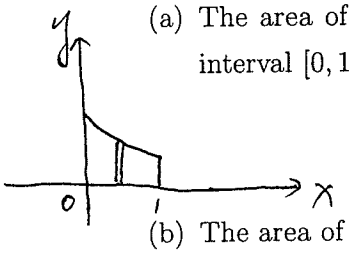
Math 220 – Quiz 12 (take-home)

Name _____

TA Section _____

- You can use calculators, textbooks, notes, other students, etc.
- This quiz is due at the beginning of class on Wednesday (MWF sections) or Thursday (TR sections).
- Do not show any work for these problems and do not evaluate any integrals – just write your answer neatly using proper notation.

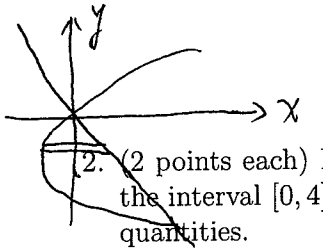
1. (2 points each) Set up, but do not evaluate, definite integrals which represent the given quantities.



(a) The area of the region bounded by the x-axis and the graph of $y = \dfrac{1}{x^2+1}$ on the interval $[0,1]$.

$$\int_0^1 \frac{1}{x^2+1}\,dx$$

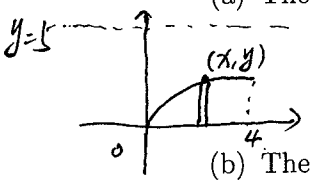
(b) The area of the region bounded by the graphs of $x + 2y = 0$ and $x = y^2 + 3y$.



$x = -2y \qquad x = y^2 + 3y$

$$\int_{-5}^{0} (-2y - y^2 - 3y)\,dy = \int_{-5}^{0} (-y^2 - 5y)\,dy$$

2. (2 points each) Let $\mathbf{R}$ be the region bounded by the x-axis and the graph of $y = \sqrt{x}$ on the interval $[0,4]$. Set up, but do not evaluate, definite integrals which represent the given quantities.

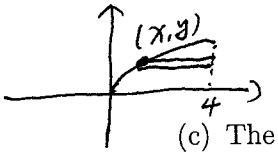


(a) The volume of the solid obtained when $\mathbf{R}$ is revolved around the line $y = 5$.

$\pi r_{out}^2 \Delta x - \pi r_{inner}^2 \Delta x = \pi \cdot 5^2 \Delta x - \pi (5-y)^2 \Delta x$

$$\int_0^4 \pi\left(5^2 - (5-y)^2\right)dx = \int_0^4 \pi\left(5^2 - (5-\sqrt{x})^2\right)dx$$

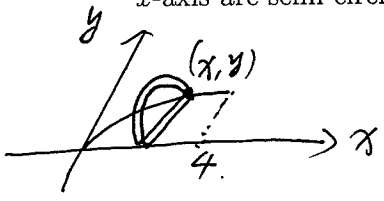
(b) The volume of the solid obtained when $\mathbf{R}$ is revolved around the y-axis.



$\pi\, 4^2 \Delta y - \pi\, x^2 \cdot \Delta y$

$$\int_0^2 \pi(4^2 - y^4)\,dy$$

(c) The volume of the solid with base $\mathbf{R}$ for which the cross-sections perpendicular to the x-axis are semi-circles.



the radius r of the semi-circle is $r = \frac{y}{2}$

$$\therefore \quad \frac{\pi r^2}{2}\Delta x = \frac{\pi\left(\frac{y}{2}\right)^2}{2}\Delta x = \frac{\pi \frac{x}{4}}{2}\Delta x$$

$$\int_0^4 \frac{\pi x}{8}\,dx$$