Math 220 – Quiz 11

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You have 12 minutes for this quiz.

1. (2 points) If $g(x) = \int_x^3 \cos\left(t^2\right) dt$, then find a formula for $g'(x)$.

$g(x) = -\int_3^x \cos(t^2)\,dt$, so by FTC Part I,

$g'(x) = \boxed{-\cos(x^2)}$

2. (2 points) At 4:00 AM, the layer of ice on Lake Mendota was 6 inches thick and its thickness was increasing at a rate of $0.2t$ inches per hour where t represents the number of hours since 4:00 AM. How thick was the ice at noon that same day?

Let $T(t)$ be thickness at time t.

Then by FTC Part II,

$$T(t) - T(0) = \int_0^8 0.2t\,dt \qquad (\text{noon} \iff t = 8)$$

$$= 0.1\,t^2\Big|_0^8 = 0.1(8^2) = 6.4$$

$T(0) = 6 \Rightarrow T(t) = 6 + 6.4 = \boxed{12.4 \text{ inches}}$.

3. (2 points each) Evaluate the following definite and indefinite integrals.

(a) $\int_0^1 \frac{10}{2x+1}\,dx$

$u = 2x+1 \Rightarrow du = 2\,dx, \quad \frac{1}{2}du = dx$

$0 \to 2(0)+1 = 1, \quad 1 \to 2(1)+1 = 3$

$= \int_1^3 \frac{10}{u}\left(\frac{1}{2}\,du\right) = 5\int_1^3 \frac{1}{u}\,du = 5(\ln 3 - \ln 1)$

$= \boxed{5\ln 3}.$

(b) $\int \frac{6x^2+5}{x}\,dx$

$= \int 6x + \frac{5}{x}\,dx = 6\int x\,dx + 5\int \frac{1}{x}\,dx$

$= 6\left(\frac{x^2}{2}\right) + 5\ln x + C$

$= \boxed{3x^2 + 5\ln x + C}.$

(c) $\int \frac{x}{e^{x^2}}\,dx$

$u = x^2 \Rightarrow du = 2x\,dx \Rightarrow \frac{1}{2}du = x\,dx$

$\int \frac{1}{e^u}\,\frac{1}{2}\,du = \frac{1}{2}\int e^{-u}\,du = -\frac{1}{2}e^{-u} + C$

$= \boxed{-\frac{1}{2}e^{-x^2} + C}$