Name ________________________________

- For this quiz you can use calculators, textbooks, notes, other students, etc.
- Be sure that all pages of your quiz are stapled together.
- There is no specific time limit, but the quiz is due at the beginning of your first discussion section meeting next week.

1. (3 points) Find the most general antiderivative of the given function. Before finding an antiderivative for part (c), you must first show how to rewrite the function in a way which leads directly to an antiderivative.

(a) $f(x) = 6\sec^2 x - 3\sec x \tan x$

$$\int (6\sec^2 x - 3\sec x \tan x)\,dx = 6\tan x - 3\sec x + C$$

(b) $f(x) = 21x^2\sqrt{x}$

$$f(x) = 21x^{5/2}$$

$$\int 21x^{5/2}\,dx = \frac{2}{7}\cdot 21x^{7/2} + C = 6x^{7/2} + C$$

(c) $f(x) = \dfrac{6x^3 - 5x^2 + 6x - 3}{x^2+1}$

Long division:

$$\begin{array}{r} 6x - 5 \\ x^2+1 \,\big)\, 6x^3 - 5x^2 + 6x - 3 \\ -(6x^3 + 6x) \\ \hline -5x^2 - 3 \\ -(-5x^2 - 5) \\ \hline 2 \end{array}$$

So $f(x) = 6x - 5 + \dfrac{2}{x^2+1}$.

$$\int \left(6x - 5 + \frac{2}{x^2+1}\right) dx = 3x^2 - 5x + 2\tan^{-1}(x) + C$$

2. (3 points) An object is thrown upward from ground level. Three seconds later it has fallen back to the ground. What is the initial velocity for this object?

$a(t) = -9.8$

$v(t) = \int -9.8\,dt = -9.8t + C$ (want to find $V(0)$).

$s(t) = \int (-9.8t + C)\,dt = -4.9t^2 + Ct + D.$

where $s(t)$ is the distance of the object from the ground at time t.

$S(0) = 0$, so $-4.9(0)^2 + C(0) + D = 0$, so $D = 0$.

So $S(t) = -4.9t^2 + Ct$.

$S(3) = 0$, so $-4.9(3)^2 + 3C = 0$, so $C = \frac{(+4.9)9}{3} = 14.7$.

So $V(t) = -9.8t + 14.7$, so $\boxed{V(0) = 14.7\,ms^{-1}}$

3. (2 points) The area between the x-axis and the graph of $f(x) = \sqrt{x}$ on the interval $[4, 9]$ can be written as a limit. Fill in the missing information in this limit.

$$AREA = \lim_{n\to\infty} \sum_{k=1}^{n} \left[\left(\sqrt{4 + \frac{5}{n}k}\right) \frac{5}{n} \right]$$

$$\Delta x = \frac{9-4}{n} = \frac{5}{n}$$

$$x_k^* = 4 + \frac{5}{n}k$$

$$\text{So Area} = \lim_{n\to\infty} \sum_{k=1}^{n} f(x_k^*)\,\Delta x$$

$$= \lim_{n\to\infty} \sum_{k=1}^{n} \left(\sqrt{4 + \frac{5}{n}k}\right) \frac{5}{n}$$

4. (2 points) A car is traveling at 60 feet per second when the driver sees a deer in the road 300 feet ahead and immediately steps on the brakes. The deer freezes and does not move from his spot in the road. I've recorded the driver's speed (in ft/sec) every two seconds starting at the time that he first stepped on the brakes and going until the time that the car finally came to a stop. **Does the car hit the deer? Explain your reasoning.**

time	0	2	4	6	8
car's speed	60	46	28	12	0

The distance the car travels is given by the integral of the speed function for the car. The car is braking, so the speed function is a decreasing function.

left

We can estimate the integral using ~~right~~-hand endpoints. As the function is decreasing, our estimate will be an over estimate, i.e. the actual distance the car travels will be less than or equal to the estimated distance.

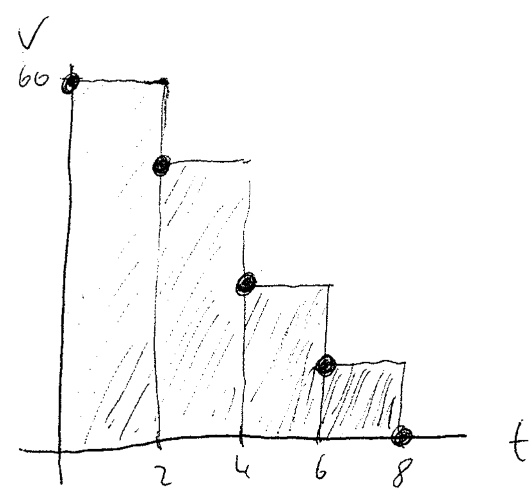
left

The estimate with ~~right~~-hand end points is

$$2(60+46+28+12)$$

$$= 2(146)$$

$$= 292 \text{ ft}$$



So the car travels a distance smaller than or equal to 292 ft, so the car does not hit the deer.