1. (8 points) Find a formula for $f'(x)$ given that $f(x) = \int_6^x 4e^{\sin(8t)}\, dt$.

$$f'(x) = 4e^{\sin(8x)}$$

2. (8 points) There are currently 500 rabbits living on an island. A biologist uses a model which predicts that the population will increase by $2t+7$ rabbits per year where t represents the number of years from today. According to the biologist's model, how many rabbits will be living on the island 10 years from now?

$$500 + \int_0^{10} (2t+7)\,dt$$

$$= 500 + \left[t^2 + 7t\right]_0^{10}$$

$$= 500 + \left[(100+70) - (0+0)\right]$$

$$= 670 \text{ rabbits}$$

3. (8 points) Evaluate the following indefinite integral.

$$\int \sec^4 x \tan x \, dx = \int \sec^3 x \sec x \tan x \, dx$$
$$= \int u^3 du \quad \begin{pmatrix} u = \sec x \\ du = \sec x \tan x \, dx \end{pmatrix}$$
$$= \frac{1}{4}u^4 + C$$
$$= \frac{1}{4}\sec^4 x + C$$

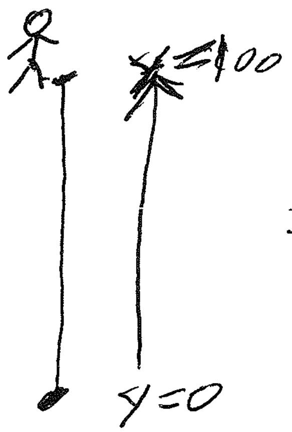
4. (8 points each) Evaluate the following definite integrals. Simplify each answer.

(a) $\int_0^{\pi/3} (12 - 4\cos x) \, dx = \left[12x - 4\sin x\right]_0^{\pi/3}$
$$= \left(12 \cdot \frac{\pi}{3} - 4\sin\left(\frac{\pi}{3}\right)\right) - (0 - 0)$$
$$= 4\pi - 4 \cdot \sqrt{3}/2$$
$$= 4\pi - 2\sqrt{3}$$

(b) $\int_0^2 10xe^{x^2} \, dx = \int_0^4 5e^u du \quad \begin{pmatrix} u = x^2 \\ du = 2x\,dx \end{pmatrix}$
$$= \left[5e^u\right]_0^4$$
$$= 5e^4 - 5$$

5. (8 points) A mountain climber is holding the top end of a 100 foot long rope which has some gear attached to the bottom. The rope weighs 0.08 pounds per foot and the gear weighs 6 pounds. How much work is required for the climber to raise the rope and gear halfway up toward his position? If you use any definite integrals to solve this problem, you can leave the definite integrals unevaluated. If you do not use any definite integrals then you must clearly explain why your method is valid.

Work = work for gear + work for top half of rope + work for bottom half of rope



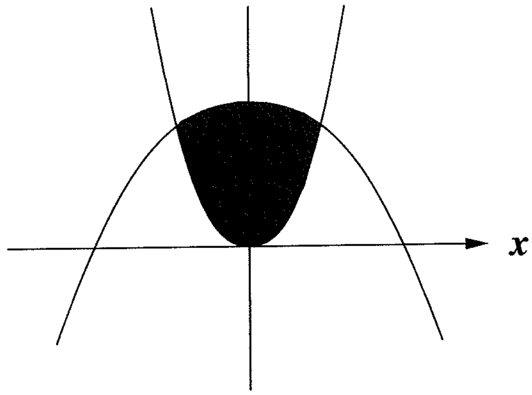
$$= 6\cdot 50 + \int_{50}^{100} 0.08(100-y)\,dy + \int_{0}^{50} 0.08(50)\,dy$$

$= 300 + 100 + 200$

$= 600$ ft-lbs

There are many other ways to solve this.

6. (8 points each) The graphs of $f(x) = x^4$ and $g(x) = 20 - x^2$ intersect at the points $(-2, 16)$ and $(2, 16)$. Let **R** be the shaded region enclosed by these two curves as shown below. Using proper mathematical terminology, write down the definite integrals which represent the following quantities. Do not evaluate these integrals.



(a) The area of **R**.

$$\int_{-2}^{2} (20 - x^2 - x^4)\,dx$$

(b) The volume of the solid obtained when **R** is revolved around the vertical line $x = 8$.

$$\int_{-2}^{2} 2\pi \underbrace{(8-x)}_{\text{radius}} \underbrace{(20 - x^2 - x^4)}_{\text{height}}\,dx$$

(c) The volume of the solid with base **R** for which the cross-sections perpendicular to the x-axis are squares.

$$\int_{-2}^{2} (20 - x^2 - x^4)^2\,dx$$

7. (8 points) Find the average value of the function $f(x) = \frac{1}{x}$ on the interval $\left[1, e^4\right]$. Simplify your answer.

$$\frac{1}{e^4-1} \int_1^{e^4} \frac{1}{x}\,dx = \frac{1}{e^4-1}\left[\ln x\right]_1^{e^4} = \frac{1}{e^4-1}\left[\ln e^4 - \ln 1\right]$$

$$= \frac{4}{e^4-1}$$

8. (6 points) Suppose that $f(x)$ is an odd function which is continuous on the interval $[-3, 3]$.

Let $g(x) = 5f(x) + 10$. Evaluate the following definite integral.

$$\int_{-3}^{3} g(x)\,dx = \int_{-3}^{3} (5f(x)+10)\,dx$$

$$= 5\int_{-3}^{3} f(x)\,dx + \int_{-3}^{3} 10\,dx$$

$$= 5 \cdot 0 + 60 = 60$$

9. (5 points) Suppose that F and F' are each differentiable (and thus continuous) everywhere and that c and d are constants. Circle the choice below which most clearly demonstrates the Fundamental Theorem of Calculus. No work needs to be shown for this problem.

(a) $\int_c^d F(t)\,dt = F'(d) - F'(c)$

(b) $\int_c^d F(t)\,dt = F'(c) - F'(d)$

(c) $\int_c^d F(t)\,dt = F(d) - F(c)$

(d) $\int_c^d F(t)\,dt = F(c) - F(d)$

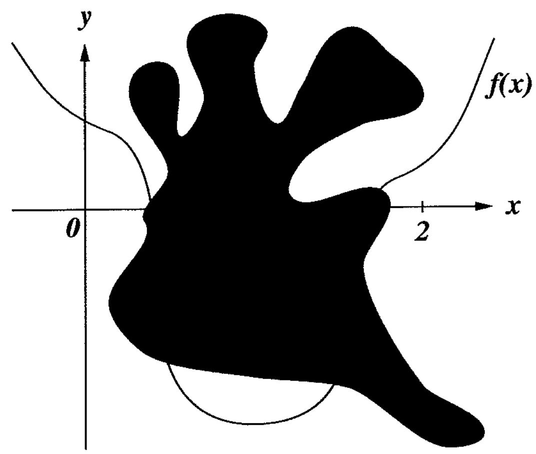
(e) $\int_c^d F'(t)\,dt = F'(d) - F'(c)$

(f) $\int_c^d F'(t)\,dt = F'(c) - F'(d)$

(g) $\int_c^d F'(t)\,dt = F(d) - F(c)$

(h) $\int_c^d F'(t)\,dt = F(c) - F(d)$

10. (9 points) Suppose that f is a polynomial whose graph is sketched below. Unfortunately, due to a terrible ink spill, only a portion of the graph can now be seen. Finish each statement by circling the correct choice and carefully explaining your reasoning. **You cannot get any credit unless you provide a correct justification for each answer.**



(a) The value of $\int_0^2 f(x)\,dx$ is

(a) negative (b) zero (c) positive (d) impossible to determine

Imagine shading in the area between the x-axis and f(x) on [0, 2]. Much more of this area lies below the x-axis than above.

(b) The value of $\int_0^2 f'(x)\,dx$ is

(a) negative (b) zero (c) positive (d) impossible to determine

$\int_0^2 f'(x)dx = f(2) - f(0)$ BY F.T.C.

$= (\text{small pos. \#}) - (\text{large pos. \#})$

(c) The value of $\int_0^2 f''(x)\,dx$ is

(a) negative (b) zero (c) positive (d) impossible to determine

$\int_0^2 f''(x)dx = f'(2) - f'(0)$ BY F.T.C.

$= (\text{pos. \#}) - (\text{neg. \#})$