MATH 220 (sections AL1, BL1, DL1, L1, L2, L3, L4) **Test 3** **Fall 2009**

1. (6 points) Evaluate the following limit. Show sufficient work to justify your answer.

$$\lim_{x\to 0} \frac{\overset{\to 0}{e^{3x}-1}}{\underset{\to 0}{8x}} \overset{L}{=} \lim_{x\to 0} \frac{3e^{3x}}{8} = \frac{3}{8}$$

2. (12 points) Evaluate the following limits. No work needs to be shown.

(a) $\lim_{x\to\infty} \frac{x^{64}}{16e^x} = 0$

$\to \infty$ slowly

$\to \infty$ quickly

(b) $\lim_{x\to\infty} \frac{8 - 2\ln x}{5x} = 0$

$-\infty$ slowly

$\to \infty$ quickly

(c) $\lim_{x\to\infty} \frac{\sqrt{x}}{\ln(9x)} = \infty$

∞ quickly

∞ slowly

3. (8 points) In order to approximate $\sqrt{26.3}$, begin with an initial estimate of $x_1 = 5$ and use Newton's Method to obtain a second estimate x_2. Simplify your final answer.

$$x = \sqrt{26.3}$$

$$x^2 = 26.3$$

$$x^2 - 26.3 = 0$$

let $f(x) = x^2 - 26.3$

$$f'(x) = 2x$$

$$x_1 = 5$$

$$x_2 = x_1 - \frac{f(x_1)}{f'(x_1)}$$

$$x_2 = 5 - \frac{f(5)}{f'(5)}$$

$$x_2 = 5 - \frac{(-1.3)}{10}$$

$$x_2 = 5 + \frac{1.3}{10}$$

$$x_2 = 5 + 0.13$$

$$\boxed{x_2 = 5.13}$$

4. (8 points) Use a linear approximation (or differential) to estimate the value of $\ln(0.95)$. Simplify your final answer.

$f(x) = \ln x$

tangent line at $x = 1$:

$f(1) = \ln 1 = 0$

Point $(1, 0)$

$f'(x) = \frac{1}{x}$

$f'(1) = 1$

$\therefore$ slope $= 1$

$y = x - 1$

$\ln x \approx x - 1$ for x near 1

So

$\ln(0.95) \approx 0.95 - 1$

$\approx \boxed{-0.05}$

5. (8 points) Find $f(x)$ given that $f'(x) = 8 + 3\sin x$ and $f(0) = 35$.

$$f(x) = 8x - 3\cos x + C$$

for $x = 0$,

$$35 = 0 - 3(1) + C$$

$$35 = -3 + C$$

$$38 = C$$

$$\boxed{f(x) = 8x - 3\cos x + 38}$$

6. (8 points) Find any antiderivative of the function $f(x) = (3x+4)(5x+2)$.

$$f(x) = 15x^2 + 26x + 8$$

$$\boxed{\text{antiderivative: } F(x) = 5x^3 + 13x^2 + 8x}$$

7. (8 points) Find the most general antiderivative of the function $f(x) = \dfrac{8x^2+6}{x}$.

$$f(x) = 8x + \frac{6}{x}$$

$$\boxed{\text{antiderivative: } F(x) = 4x^2 + 6\ln(|x|) + C}$$

8. (6 points) Evaluate the following limit.

$$\lim_{n\to\infty}\sum_{i=1}^{n}\frac{6i}{n^2}$$

$$\lim_{n\to\infty}\sum_{i=1}^{n}\frac{6i}{n^2} = \lim_{n\to\infty}\frac{6}{n^2}\sum_{i=1}^{n} i$$

$$= \lim_{n\to\infty}\frac{6}{n^2}\left(\frac{n(n+1)}{2}\right)$$

$$= \lim_{n\to\infty}\frac{6n^2+6n}{2n^2}$$

$$= \lim_{n\to\infty}\left(3+\frac{3}{n}\right)$$

$$= 3$$

9. (6 points) The value of the definite integral $\int_8^{14}\ln x\,dx$ can be written as a limit. Fill in the missing information in this limit.

$$\int_8^{14}\ln x\,dx = \lim_{n\to\infty}\sum_{i=1}^{n}\left(\ln\left(8+i\left(\frac{6}{n}\right)\right)\cdot\frac{6}{n}\right)$$

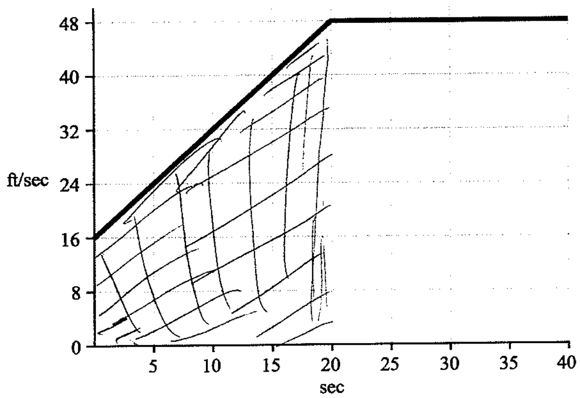
$f(x) = \ln x$

$$\Delta x = \frac{14-8}{n} = \frac{6}{n}$$

$$x_i^* = 8 + i\Delta x = 8 + i\left(\frac{6}{n}\right)$$

$$\int_8^{14}\ln x\,dx = \lim_{n\to\infty}\sum_{i=1}^{n} f(x_i^*)\cdot\Delta x$$

10. (10 points) An object's velocity in feet per second is given by the graph for the time interval shown. What is the exact total distance the object travels during the first 20 seconds of this interval?



$d = \int_0^{20} \text{velocity}\, dt$ = shaded area = $\boxed{640 \text{ ft}}$

11. (12 points) Suppose that f is integrable on the interval $[4, 10]$. Given that $\int_4^{10} f(x)\,dx = 10$ and $\int_4^6 f(x)\,dx = 3$, evaluate the following definite integrals.

(a) $\int_6^4 f(x)\,dx = -\int_4^6 f(x)\,dx = -3$

(b) $\int_6^6 f(x)\,dx = 0$

(c) $\int_6^{10} f(x)\,dx = \int_4^{10} f(x)\,dx - \int_4^6 f(x)\,dx$

$= 10 - 3$

$= 7$

12. (8 points) A piece of wire 6 feet long is to be cut into two pieces. One piece of length x will be bent to form a circle and the other piece of length $6-x$ will be bent to form a square. If the wire is cut so that the combined total area enclosed by the circle and the square is a minimum, then what is the value of x ?

circumference $= x$

$2\pi r = x$

$r = \frac{x}{2\pi}$

perimeter $= 6-x$

$s =$ side length $= \frac{6-x}{4}$

$$\text{Area} = \pi r^2 + s^2$$

$$A = \pi\left(\frac{x}{2\pi}\right)^2 + \left(\frac{6-x}{4}\right)^2$$

$$A = \frac{x^2}{4\pi} + \frac{36-12x+x^2}{16}$$

$$A' = \frac{2x}{4\pi} + \frac{0-12+2x}{16}$$

Set $A'=0$ and solve for x

$$0 = \frac{2x}{4\pi} + \frac{2x-12}{16}$$

$$0 = \frac{x}{2\pi} + \frac{x}{8} - \frac{3}{4}$$

$$\frac{3}{4} = x\left(\frac{1}{2\pi} + \frac{1}{8}\right)$$

$$x = \frac{3/4}{\left(\frac{1}{2\pi} + \frac{1}{8}\right)} = \boxed{\frac{6\pi}{4+\pi}}$$

Values of A'

$- - - \; 0 \; + + +$

$0 \qquad \frac{6\pi}{4+\pi} \qquad 6$