SOLUTIONS

1. (5 points) If the point $(3, -6)$ is on the graph of a one-to-one function f, then which one of the following points must be on the graph of f^{-1} ?

   (a) $(-6, 3)$
   (b) $(-6, -3)$
   (c) $(3, 6)$
   (d) $(-3, 6)$
   (e) $(-3, -6)$
   (f) $(6, 3)$
   (g) $(6, -3)$

$f(a) = b \Leftrightarrow f^{-1}(b) = a$

(a,b) ON GRAPH OF f $\Longleftrightarrow$ (b,a) ON GRAPH OF f^{-1}

2. (5 points) If the point $(3, -6)$ is on the graph of an odd function f, then which one of the following points must also be on the graph of f ?

   (a) $(-6, 3)$
   (b) $(-6, -3)$
   (c) $(3, 6)$
   (d) $(-3, 6)$
   (e) $(-3, -6)$
   (f) $(6, 3)$
   (g) $(6, -3)$

$f(3) = -6$

SINCE f IS ODD,

$f(-3) = -(-6) = 6$

SO $(-3, 6)$ IS ON THE GRAPH OF f

3. (5 points) If the point $(3, -6)$ is on the graph of an even function f, then which one of the following points must also be on the graph of f ?

   (a) $(-6, 3)$
   (b) $(-6, -3)$
   (c) $(3, 6)$
   (d) $(-3, 6)$
   (e) $(-3, -6)$
   (f) $(6, 3)$
   (g) $(6, -3)$

$f(3) = -6$

SINCE f IS EVEN,

$f(-3) = -6$

SO $(-3, -6)$ IS ON THE GRAPH OF f

4. (5 points) What is the domain of the function $f(x) = \sqrt{x+9} + \sqrt{x+2}$?

(a) $(-\infty, -9]$

(b) $(-\infty, -2]$

(c) $(-\infty, 2]$

(d) $(-\infty, 9]$

(e) $(-\infty, \infty)$

(f) $[-9, \infty)$

(g) $[-2, \infty)$

(h) $[2, \infty)$

(i) $[9, \infty)$

(j) $[-9, -2]$

(k) $[2, 9]$

$x + 9 \geq 0$ AND $x + 2 \geq 0$

$$\underbrace{x \geq -9 \text{ AND } x \geq -2}_{x \geq -2}$$

5. (5 points) Given that $7^t = 2$, what is the exact value of t ?

(a) $2/7$

(b) $7/2$

(c) $\ln(2/7)$

(d) $\ln(7/2)$

(e) $\dfrac{\ln 2}{\ln 7}$

(f) $\dfrac{\ln 7}{\ln 2}$

(g) $\dfrac{2}{\ln 7}$

(h) $\dfrac{7}{\ln 2}$

(i) $\dfrac{\ln 2}{7}$

(j) $\dfrac{\ln 7}{2}$

$\ln(7^t) = \ln(2)$

$t \cdot \ln(7) = \ln(2)$

$t = \dfrac{\ln(2)}{\ln(7)}$

6. (8 points) Find a formula for $f^{-1}(x)$ given that $f(x) = \ln\left(\frac{x-8}{5}\right)$.

LET $y = f^{-1}(x)$

THEN $f(y) = x$

$\ln\left(\frac{y-8}{5}\right) = x$

$\frac{y-8}{5} = e^x$

$y - 8 = 5e^x$

$y = 8 + 5e^x$

$f^{-1}(x) = 8 + 5e^x$

7. (8 points) Simplify the following expression.

APPROACH 1

$\sec\left(\tan^{-1}(3)\right) = \sqrt{\sec^2(\tan^{-1}(3))}$

$= \sqrt{\tan^2(\tan^{-1}(3)) + 1}$

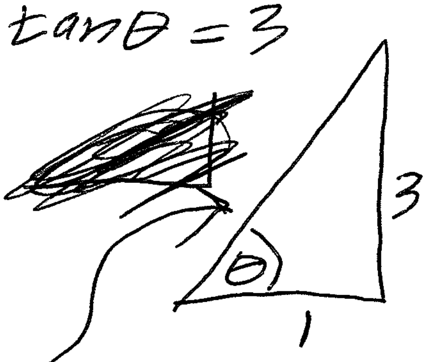
$= \sqrt{3^2 + 1}$

$= \sqrt{10}$

APPROACH 2

LET $\theta = \tan^{-1}(3)$

$\tan\theta = 3$



$\sqrt{10}$ FROM PYTHAGOREAN THEOREM

$1^2 + 3^2 = c^2 \Rightarrow c = \sqrt{10}$

$\sec(\tan^{-1}(3)) = \sec\theta$

$= \frac{1}{\cos\theta}$

$= \frac{1}{1/\sqrt{10}}$

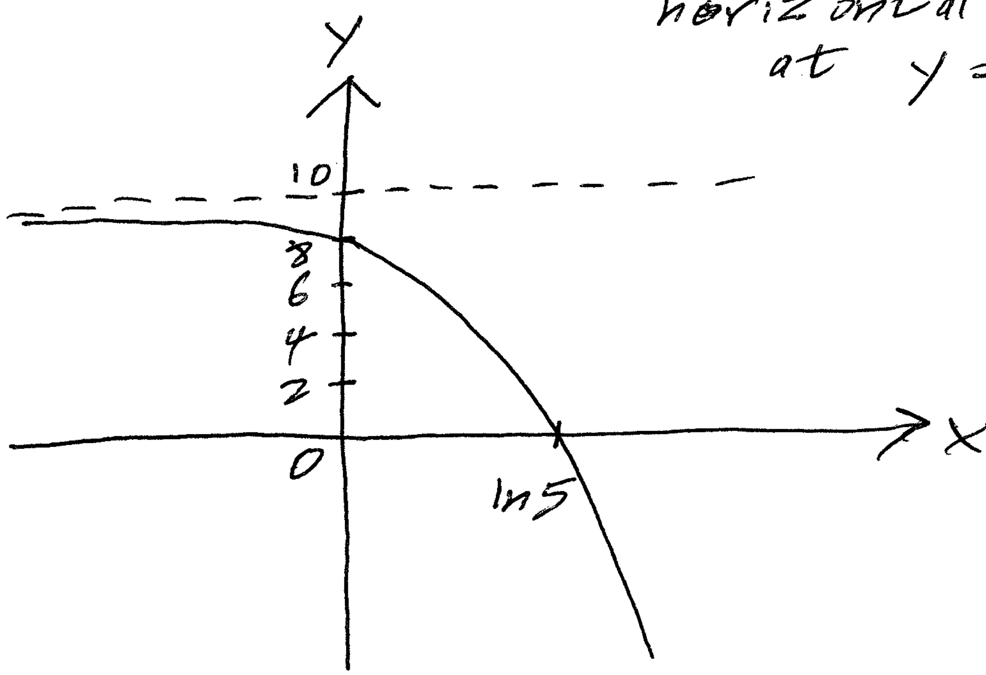
$= \sqrt{10}$

8. (8 points) Given that $f(x) = x^2 + 10$ and $g(x) = x^2 + 5$, find a formula for $(g \circ f)(x)$. You do not need to simplify your answer.

$$(g \circ f)(x) = g(f(x))$$
$$= g(x^2+10)$$
$$= (x^2+10)^2 + 5$$

9. (8 points) Carefully sketch the graph of $f(x) = 10 - 2e^x$. Be sure to find the exact value of each horizontal and vertical intercept. The locations of any intercepts or asymptotes should be accurately shown on your graph.

horizontal asymptote at $y = 10$



y-int: $f(0) = 10 - 2e^0 = 8$

x-int: $0 = 10 - 2e^x$

$2e^x = 10$

$e^x = 5$

$x = \ln 5$

10. (5 points each) Evaluate the following limits. When the limit is infinite be sure to state whether it is ∞ or $-\infty$.

(a) $\lim_{x\to 0} \frac{4x+1}{2-x}$ $= \frac{4(0)+1}{2-0} = \frac{1}{2}$

since $\frac{4x+1}{2-x}$ is continuous at 0

(b) $\lim_{x\to 2^+} \frac{4x+1}{2-x}$ $= -\infty$

Since numerator $\to 9$
and denominator $\to 0$ through negative values

(c) $\lim_{x\to\infty} \frac{4x+1}{2-x}$ $= \lim_{x\to\infty} \frac{(4x+1)\cdot\frac{1}{x}}{(2-x)\cdot\frac{1}{x}}$

$= \lim_{x\to\infty} \frac{4+\frac{1}{x}}{\frac{2}{x}-1}$

$= \frac{4+0}{0-1}$

$= -4$

(d) $\lim_{t\to\infty} \frac{\sin(4t)}{t}$ $= 0$

$-1 \le \sin(4t) \le 1$

$-\frac{1}{t} \le \frac{\sin(4t)}{t} \le \frac{1}{t}$ (FOR $t > 0$)

since $\lim_{t\to\infty} \frac{-1}{t} = \lim_{t\to\infty} \frac{1}{t} = 0$, WE HAVE

$\lim_{t\to\infty} \frac{\sin(4t)}{t} = 0$ BY THE SQUEEZE THEOREM

(e) $\lim_{x\to\infty} 5e^{1/x^2}$ $= 5e^0 = 5$

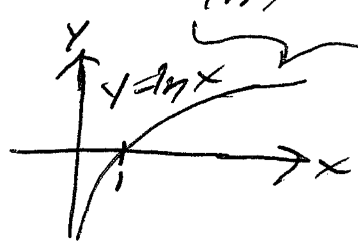
SINCE $\frac{1}{x^2} \to 0$ AS $x \to \infty$

(f) $\lim_{x\to 1^-} \frac{\sqrt{x}}{\ln x}$ (hint – it may help to think about the graph of $\ln x$)

$\lim_{x\to 1^-} \frac{\sqrt{x}}{\ln x} = -\infty$

SINCE $\sqrt{x} \to 1$ AS $x \to 1^-$

AND $\ln x \to 0$ THROUGH NEGATIVE VALUES AS $x \to 1^-$



(g) $\lim_{h\to 0} \frac{\sqrt{25+h}-5}{h}$ $= \lim_{h\to 0} \frac{(\sqrt{25+h}-5)(\sqrt{25+h}+5)}{h(\sqrt{25+h}+5)}$

$$= \lim_{h\to 0} \frac{h}{h(\sqrt{25+h}+5)}$$

$$= \lim_{h\to 0} \frac{1}{\sqrt{25+h}+5}$$

$$= \frac{1}{\sqrt{25}+5}$$

$$= \frac{1}{10}$$

11. (8 points) What value of c makes the following function continuous on $(-\infty, \infty)$?

$$f(x) = \begin{cases} x^2+5 & \text{for} \quad x < 2 \\ 5x+c & \text{for} \quad x \geq 2 \end{cases}$$

x^2+5 IS CONTINUOUS FOR $x<2$

AND $5x+c$ IS CONTINUOUS FOR $x>2$

AT $x=2$ WE MUST CHECK THAT

$\lim_{x\to 2} f(x) = f(2)$

① $f(2) = 5(2)+c = 10+c$

② $\lim_{x\to 2^-} f(x) = \lim_{x\to 2^-} (x^2+5) = 9$

③ $\lim_{x\to 2^+} f(x) = \lim_{x\to 2^+} (5x+c) = 10+c$

SOLVING $10+c=9$ GIVES $c=-1$