1. (4 points) The dynamical system shown has a stable equilibrium point at \((p^*, q^*) = (100, 75)\). Given that \(p(0) = 15\) and \(q(0) = 20\), determine the eventual rate at which \(p\) approaches equilibrium. Show all calculations you made to find the rate.

\[
p(t) = 0.7p(t-1) + 0.2q(t-1) + 15
\]

\[
q(t) = 0.1p(t-1) + 0.6q(t-1) + 20
\]

\[
\frac{p(1)-100}{p(0)-100} \approx 0.8294117647
\]

\[
\frac{p(2)-100}{p(1)-100} \approx 0.8177304965
\]

\[
\frac{p(3)-100}{p(2)-100} \approx 0.8108412836
\]

\[
\frac{p(4)-100}{p(3)-100} \approx 0.809842438126
\]

\[
\frac{p(5)-100}{p(4)-100} \approx 0.809700022157
\]

\[
\frac{p(6)-100}{p(5)-100} \approx 0.809670015081
\]

\[
\lim_{t \to \infty} \frac{p(t)-100}{p(t-1)-100} = 0.8
\]

Eventually, \(p\) gets 20% closer to its equilibrium each time period.
2. (6 points) For the following discrete dynamical systems,

- Find an explicit formula for the given function.
- Determine the equilibrium value, or state that no equilibrium value exists.
- For systems with an equilibrium value, state whether the equilibrium value is stable or unstable.
- For systems with an equilibrium value, determine the rate at which the function approaches or moves away from equilibrium.

(a) \(q(t + 1) = q(t) - 3\) and \(q(0) = 60\)

\[ q(t) = -3t + 60 \]

\text{NO EQUILIBRIUM VALUE}

\text{SINCE LINEAR WITH NONZERO SLOPE}
(b) $u(n) = 1.1u(n-1)$ and $u(0) = 40$

- $u(n) = 40 \cdot (1.1)^n$
- $u^* = 0$

As an **unstable equilibrium value** since $1.1 > 1$

$$\lim_{n \to \infty} \frac{u(n) - 0}{u(n-1) - 0} = \lim_{n \to \infty} \frac{1.1u(n-1)}{u(n-1)}$$

$$= \lim_{n \to \infty} 1.1$$

$$= 1.1$$

So $u$ moves 10% further away from equilibrium each time period.
(c) \( P(t) = 0.92P(t-1) + 16 \) and \( P(0) = 25 \)

\[
P^* = 0.92P^* + 16
\]

\[
0.08P^* = 16
\]

\[
P^* = \frac{16}{0.08} = 200
\]

\[
P(t) = c(0.92)^t + 200
\]

\[
25 = c(0.92)^0 + 200
\]

\[
25 = c + 200
\]

\[
c = -175
\]

\[
P(t) = -175(0.92)^t + 200
\]

\[
P^* = 200
\]

\[P^* = 200 \text{ is a stable equilibrium since } -1 < 0.92 < 1\]

\[
\lim_{t \to \infty} \left(\frac{P(t)-200}{P(t-1)-200}\right) = \lim_{t \to \infty} \left(\frac{0.92P(t-1) + 16 - 200}{P(t-1) - 200}\right)
\]

\[
= \lim_{t \to \infty} \left(\frac{0.92P(t-1) - 134}{P(t-1) - 200}\right)
\]

\[
= \lim_{t \to \infty} \left(\frac{0.92(P(t-1) - 200)}{P(t-1) - 200}\right)
\]

\[
= 0.92
\]

P moves 8% closer to its equilibrium each time around.