Name: SOLUTIONS

- Do not open this test booklet until told to do so.
- Turn off all cell phones.
- Show sufficient work to justify each answer.
- You are not allowed to borrow another student’s calculator during the test.

Do not write below this line

#1 (5 points) ___________ #7 (16 points) ___________
#2 (6 points) ___________ #8 (15 points) ___________
#3 (6 points) ___________ #9 (12 points) ___________
#4 (6 points) ___________ #10 (16 points) ___________
#5 (6 points) ___________
#6 (12 points) ___________

TOTAL (100 points) ___________

Test 1 ___________ Test 2 ___________ Test 3 ___________ Total ___________

If you skip the final exam, your course grade will be ___________________________
1. (5 points) Find an explicit solution to the initial value problem.

\[ \frac{dP}{dt} = 5 \quad \text{and} \quad P(3) = 45 \]

\[ P = 5t + C \]
\[ 45 = 5 \cdot 3 + C \]
\[ C = 30 \]
\[ P = 5t + 30 \]

2. (6 points) Find an explicit solution to the initial value problem.

\[ \frac{dw}{dt} = 3w \quad \text{and} \quad w(0) = 20 \]

\[ \int \frac{dw}{w} = \int 3 \, dt \]
\[ \ln w = 3t + C \]
\[ \ln(20) = 3 \cdot 0 + C \]
\[ C = \ln 20 \]
\[ \ln w = 3t + \ln 20 \]
\[ w = e^{3t} \cdot e^{\ln 20} \]
\[ w = 20e^{3t} \]

or just memorize

3. (6 points) Find an explicit solution to the initial value problem.

\[ \frac{dw}{dt} = 4t \quad \text{and} \quad w(0) = 30 \]

\[ w = 2t^2 + C \]
\[ 30 = 2 \cdot 0^2 + C \]
\[ C = 30 \]
\[ w = 2t^2 + 30 \]
4. (6 points) Find an explicit solution to the initial value problem.

\[ \frac{dy}{dx} = -8e^{-4x} \quad \text{and} \quad y(0) = 10 \]

\[ y = 2e^{-4x} + C \]

\[ 10 = 2e^{0} + C \]

\[ 10 = 2 + C \]

\[ C = 8 \]

\[ y = 2e^{-4x} + 8 \]

5. (6 points) Find an explicit solution to the initial value problem.

\[ \frac{dy}{dx} = \frac{e^x + 2x}{y^2} \quad \text{and} \quad y(0) = 3 \]

\[ \int y^2 \, dy = \int (e^x + 2x) \, dx \]

\[ \frac{1}{3}y^3 = e^x + x^2 + C \]

\[ \frac{1}{3}(3)^3 = e^0 + 0^2 + C \]

\[ 9 = 1 + C \]

\[ C = 8 \]

\[ \frac{1}{3}y^3 = e^x + x^2 + 8 \]

\[ y^3 = 3e^x + 3x^2 + 24 \]

\[ y = \sqrt[3]{3e^x + 3x^2 + 24} \]
6. (12 points) Suppose that a fish population grows logistically with an intrinsic growth rate of 30% and a carrying capacity of 600.

(a) Determine a discrete dynamical system to model this fish population.

\[ P(t) = P(t-1) + 0.3P(t-1) \left(1 - \frac{P(t-1)}{600}\right) \]

(b) Determine the maximum interval of stability for this fish population.

0 is an unstable equilibrium value. 600 is a stable equilibrium value. The maximum interval of stability for \( r^* = 0.3 \) is \((0, 2600)\).

You can find this by trying different starting values on the calculator or by solving

\[ 0 = P + 0.3P \left(1 - \frac{P}{600}\right) \]

to get \( P = 0 \) or \( P = 2600 \).
7. (16 points) For Nancy's metabolism, the dynamical system modeling the elimination of alcohol is given by

\[ a(n) = a(n-1) - \frac{9.5a(n-1)}{4 + a(n-1)} + d \]

where \( a(n) \) is the amount of alcohol (in grams) in her bloodstream after \( n \) hours of drinking \( d \) grams of alcohol per hour.

(a) How many grams of alcohol per hour can Nancy drink if at the end of a 4 hour party she is to have 40 grams of alcohol in her bloodstream? Begin with \( a(0) = 0 \) and give your answer correct to one place after the decimal.

\[ \text{Try different values for } d \text{ on the calculator to eventually obtain } d = 16.05 \text{ gives } a(4) \approx 39.973 \]
\[ \text{and } d = 16.06 \text{ gives } a(4) \approx 40.011 \]
\[ \text{so } 16.05 < d < 16.06 \]
\[ \text{Thus } d \approx 16.1 \text{ grams} \]

(b) Compute the equilibrium amount of alcohol in Nancy's bloodstream if she drinks 9 grams of alcohol per hour.

\[ a^* = a^* - \frac{9.5a^*}{4+a^*} + 9 \]

\[ \frac{9.5a^*}{4+a^*} = 9 \]

\[ 9.5a^* = 36 + 9a^* \]

\[ 0.5a^* = 36 \]

\[ a^* = 72 \text{ grams} \]
8. (15 points) There are currently 200 deer, but the population is expected to grow exponentially by 4% each year from now on.

(a) Sketch a graph of growth rate as a function of population.

(b) Sketch a graph of population as a function of time.

(c) Sketch a graph of yearly growth as a function of population.
9. (12 points) Suppose that a bird population grows logistically with an intrinsic growth rate of 25% and a carrying capacity of 10,000.

(a) Sketch a graph of growth rate as a function of population.

(b) Sketch a graph of population as a function of time.
10. (16 points) The natural yearly growth \( g \) in a population is a function of the population size \( u \) (in thousands) and is shown in the following graph.

(a) Estimate the stable equilibrium population if there is a constant yearly harvest of 300.

\[ 55000 \]

(b) Estimate the minimum viable population if there is a constant yearly harvest of 300.

\[ 25000 \]

(c) Estimate the maximum constant sustainable yearly harvest.

\[ 400 \]

(d) Approximate the percent of the population that should be harvested each year to maximize the sustainable harvest.

\[
\frac{400}{40000} = 0.01 = 1\%\]