1. Suppose you borrow $120,000 at an 8.4\% \text{ annual interest rate compounded monthly to be paid back in monthly payments of $1800.}

(a) Write down a discrete dynamical system with initial condition to represent the balance of the loan just after each month’s payment.

Solution:
Let $u(n)$ represent the amount owed $n$ years after obtaining the loan. Then
\[ u(n) = u(n - 1) + (0.084/12)u(n - 1) - 1800 \quad \text{and} \quad u(0) = 120,000 \]
or in simplified form
\[ u(n) = 1.007u(n - 1) - 1800 \quad \text{and} \quad u(0) = 120,000 \]

(b) How many months will it take to pay back the loan?

Solution:
We find that $u(90) = 206.51$ and $u(91) = -1592.04$
It will take [91 months] but if each payment is $1800, then we have overpaid the loan by the $1592.04.

(c) The last payment will be a bit different than each of the preceding monthly payments. What will be the amount of this last payment?

Solution:
The first 90 payments will be for $1800 each, but the last (91st) payment will be for $1800 - 1592.04 = $207.96.$

2. Find the equilibrium value for the following dynamical system.
\[ u(n) = 0.9u(n - 1) - 3.5 \]

Solution:
Solving the equation $E = 0.9E - 3.5,$ we get $E = -35,$ so the equilibrium value for $u$ is $-35.$

3. Find the equilibrium value for the following dynamical system.
\[ u(n) = 1.25u(n - 1) - 6.1 \]

Solution:
Solving the equation $E = 1.25E - 6.1,$ we get $E = 24.4,$ so the equilibrium value for $u$ is 24.4.
4. Find the equilibrium point for the following dynamical system.

\[
\begin{align*}
    u(n) &= 2u(n - 1) + v(n - 1) + 3 \\
    v(n) &= 4u(n - 1) - v(n - 1) + 6
\end{align*}
\]

Solution:
Solving the equations \( E = 2E + F + 3 \) and \( F = 4E - F + 6 \), we get \( E = -2 \) and \( F = -1 \), so the equilibrium point for \((u, v)\) is \((-2, -1)\).