1. (10 points) The population of an animal species is given by \( P(t) = 5t^2 + 30t + 2500 \) where \( t \) represents the number of months since January 1, 2004. How quickly is the population growing 1 year later?

2. (12 points) Using \( P \) for your dependent variable, \( t \) for your independent variable, and \( k, r, m, \) or \( C \) for any necessary constants, write down the general form for a differential equation which models each of the following types of growth.

(a) logistic growth

(b) linear growth

(c) exponential growth
3. (5 points) If \( f(x) = 3x^5 + 10 \), then

\[ f'(x) = \]

4. (5 points) If \( y = \frac{2}{x^3} \), then

\[ \frac{dy}{dx} = \]

5. (5 points) If \( w = 2e^{3t} \), then

\[ \frac{dw}{dt} = \]

6. (5 points) If \( h = e^{-t} \), then

\[ \frac{dh}{dt} = \]
7. (10 points) Suppose that 500 trout are released into a man-made lake which had no trout before. Further suppose that the trout population, \( P \), grows logistically according to the following differential equation where \( t \) represents the number of years since the initial release of the trout.

\[
\frac{dP}{dt} = 0.1P \left(1 - \frac{P}{2500}\right), \quad P(0) = 500
\]

(a) As a percentage, what is the intrinsic growth rate of this trout population?

(b) What is the carrying capacity for this trout population?

(c) Sketch a rough graph of this trout population being sure to show any long-term behavior.
8. (12 points) Suppose \( y \) is a function of \( t \) which satisfies the differential equation

\[
\frac{dy}{dt} = \frac{3(y - 6)(y - 24)}{20}
\]

(a) For which values of \( y \) is the quantity \( y \) increasing?

(b) For which values of \( y \) is the quantity \( y \) decreasing?

(c) For which values of \( y \) is the quantity \( y \) in equilibrium? Determine whether each of these equilibrium values is stable or unstable.
9. (14 points) Suppose that 100 rabbits are released on an island that had no previous rabbits. Let \( R \) denote the rabbit population \( t \) months after they were released. The rabbit population grows at a rate which is proportional to the population size itself, where the constant of proportionality is 0.05 (i.e. a continuous growth rate of 5% per month).

(a) Write down a differential equation with initial condition for the growth of this rabbit population.

(b) Find a formula for \( R \) as a function of \( t \).

(c) Use your formula to determine the number of rabbits on the island 12 months after they were released.
10. (10 points) Find a formula for $y$ as a function of $t$ in the following initial value problems.

(a) $\frac{dy}{dt} = 6t^2 + 5, \quad y(0) = 8$

(b) $\frac{dy}{dt} = \frac{2t}{3y^2}, \quad y(0) = 5$
11. (12 points) Given the following initial value problem, use Euler’s Method with $\Delta t = 2$ to estimate $y(6)$.

\[
\frac{dy}{dt} = \sqrt{y}, \quad y(0) = 10
\]