1. (10 points) Complete the list below by writing down the Maclaurin series and the interval of convergence for each function.

\[ e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \frac{x^4}{4!} + \cdots \quad \text{for } -\infty < x < \infty \]

\[ \ln(1 + x) = \]

\[ \cos x = \]

\[ \sin x = \]

\[ \tan^{-1} x = \]

\[ \frac{1}{1 - x} = \]
2. (6 points) Find the Maclaurin series and the interval of convergence for \( f(x) = \frac{1}{1 - 3x} \)

3. (6 points) Find the Maclaurin series and the interval of convergence for \( f(x) = x^2 e^{-x} \)
4. (6 points) Let $P_0(x)$, $P_1(x)$, and $P_2(x)$ be the Maclaurin polynomials of degree 0, 1, and 2, respectively, for the function $f(x)$. Sketch plausible graphs for $P_0(x)$, $P_1(x)$, and $P_2(x)$. 

![Graphs of $P_0(x)$, $P_1(x)$, and $P_2(x)$]

$P_0(x)$

$P_1(x)$

$P_2(x)$
5. (6 points) Does the following series converge or diverge? Thoroughly justify your answer. If it is a convergent series, you will receive one bonus point by computing its sum.

\[ \sum_{k=1}^{\infty} \frac{2}{3k} \]

6. (6 points) Does the following series converge or diverge? Thoroughly justify your answer. If it is a convergent series, you will receive one bonus point by computing its sum.

\[ \sum_{k=1}^{\infty} \frac{k^2}{k^5 + 1} \]
7. (6 points) Does the following series converge or diverge? Thoroughly justify your answer. If it is a convergent series, you will receive one bonus point by computing its sum.

$$\sum_{k=1}^{\infty} \frac{2^k - 1}{3^k}$$

8. (6 points) Does the following series converge or diverge? Thoroughly justify your answer. If it is a convergent series, you will receive one bonus point by computing its sum.

$$\sum_{k=2}^{\infty} \left( \frac{k-1}{k} \right)^k$$
9. (6 points) Does the following series converge or diverge? Thoroughly justify your answer. If it is a convergent series, you will receive one bonus point by computing its sum.

\[
\sum_{k=1}^{\infty} \left( \frac{1}{2^k+1} - \frac{1}{2^{k+1}} \right)
\]

10. (6 points) Does the following series converge or diverge? Thoroughly justify your answer. If it is a convergent series, you will receive one bonus point by computing its sum.

\[
\sum_{k=1}^{\infty} \frac{2^k}{k!}
\]
11. (6 points) Does the following series converge or diverge? Thoroughly justify your answer.
If it is a convergent series, you will receive one bonus point by computing its sum.

\[ 3 - \frac{3}{2} + \frac{3}{4} - \frac{3}{8} + \frac{3}{16} - \frac{3}{32} + \cdots \]

12. (6 points) Does the following series converge or diverge? Thoroughly justify your answer.
If it is a convergent series, you will receive one bonus point by computing its sum.

\[ 1 + \frac{1}{2\sqrt{2}} + \frac{1}{3\sqrt{3}} + \frac{1}{4\sqrt{4}} + \frac{1}{5\sqrt{5}} + \frac{1}{6\sqrt{6}} + \cdots \]
13. (6 points) Approximate the quantity $\frac{1}{e}$ with an error of less than 0.1 by using an appropriate Maclaurin series. Specifically show why you know that the error is less than 0.1.

14. (6 points) Find the 3rd Taylor polynomial for $f(x) = \sqrt{x}$ about $x = 1$. 
15. (6 points) Find the interval of convergence for the following power series. Thoroughly justify your answer.

\[ 1 + 2x + 4x^2 + 8x^3 + 16x^4 + 32x^5 + 64x^6 + \ldots \]

16. (6 points) Find the interval of convergence for the following power series. Thoroughly justify your answer.

\[ \sum_{k=1}^{\infty} \frac{(x - 3)^k}{k \cdot 2^k} \]
• (2 bonus points) Euler showed that

\[ \frac{\pi^2}{6} = 1 + \frac{1}{2^2} + \frac{1}{3^2} + \frac{1}{4^2} + \frac{1}{5^2} + \frac{1}{6^2} + \cdots \]

Use this fact to find the sum of the series

\[ 1 + \frac{1}{3^2} + \frac{1}{5^2} + \frac{1}{7^2} + \frac{1}{9^2} + \frac{1}{11^2} + \cdots \]